AN INVESTIGATION OF THE ROLL-YAW COUPLINGS IN A SURFACE-TO-AIR GUIDED MISSILE

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ABSTRACT

The system studied is a surface-to-air guided missile whose aerodynamic and control characteristics can be represented by a set of linear differential equations. The objects of the study are to select a set of control system gains which will give specified performance of the missile within the roll and yaw subsystems and at the same time to minimize the coupling effects whereby motion about one of the axes causes dynamic response about the other axis.

The system is studied analytically using the mathematics of control system synthesis and design. Analytic investigations are paralleled by system simulation on a REAC analogue computer.

Using the above methods, gains are selected which give specified response in the roll stabilization and yaw control subsystems and suitably damped response due to coupled motions between systems. The study also investigates the possibilities of compensation to attenuate or eliminate coupled response.

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OBJECT

The object of this paper is to analyze a surface-to-air missile using equations which describe its aerodynamic and control system responses; to select values for control system parameters which will give specified response; and to investigate the possibility of compensation which will eliminate coupled response effects between subsystems.

CHAPTER 1

INTRODUCTION

1.1 Physical Description of the Missile System

The missile studied is a surface launched antiaircraft weapon. It is a homing missile using proportional navigation. Thus the missile borne receiving antenna tracks the target and the guidance system continually measures the angular rate of change of the line of sight to the target. The guidance computer generates command signals for yaw and pitch autopilot systems. The autopilot systems react by changing the missile heading at a rate proportional to the rate of change of the line of sight. Because of cruciform missile shape, the yaw and pitch autopilots have the same form. Polarization of the receiving antenna and the insertion of additional autopilot commands in earth coordinates require roll stabilization of the missile.

The general control problems of the autopilot, in response to commands generated by the guidance computer, may be stated as:

1. An output acceleration in yaw equal to the command acceleration in yaw with as fast a response as possible.
2. Same response as above, but in pitch.
3. Maintenance of roll stabilization (zero roll angle) in the presence of induced roll torques.

1.2 System Equations

The coordinate system to be used is shown in Figure 1.1. The axes are centered at c. g. and rotate with the missile airframe.

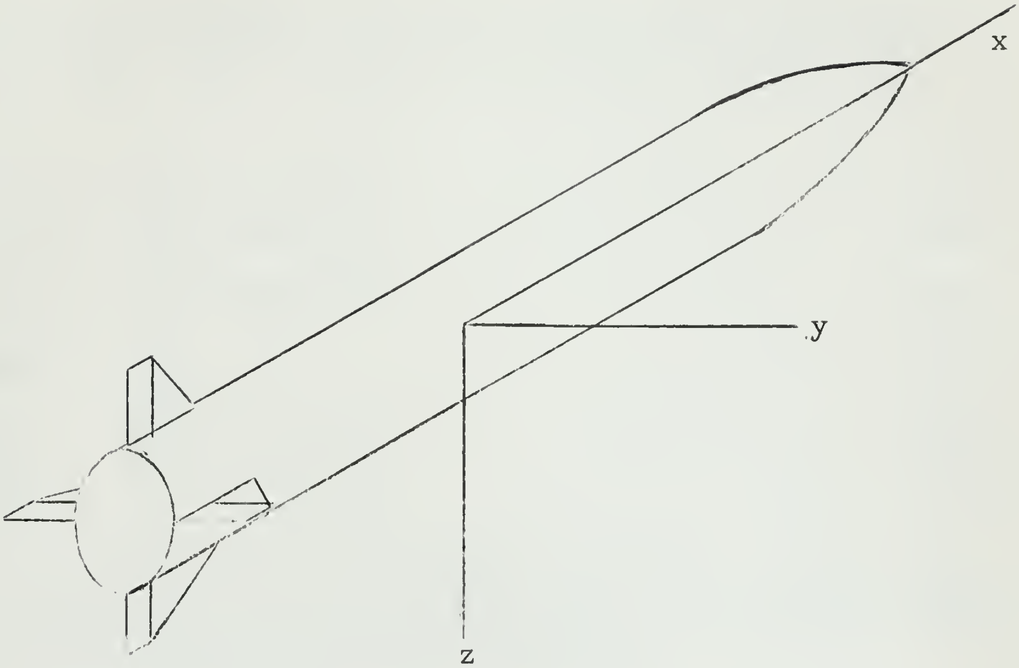

Fig. 1.1 Missile Axis Orientation

Consider $\hat{i}$, $\hat{j}$, $\hat{k}$ to be unit vectors along the x, y, z axes respectively. Define the following:

$\overline{F}$	$\equiv$	applied force $\equiv F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$	(1.1)
$\overline{V}$	$\equiv$	velocity of the missile $\equiv u\hat{i} + v\hat{j} + w\hat{k}$	(1.2)
$\overline{\omega}$	$\equiv$	angular velocity of the coordinate system	
	$\equiv$	$p\hat{i} + q\hat{j} + r\hat{k}$	(1.3)
m	$\equiv$	mass of missile	(1.4)
$\overline{M}$	$\equiv$	applied moment $\equiv M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$	(1.5)
$\overline{H}$	$\equiv$	angular momentum of missile	
	$\equiv$	$I_x p\hat{i} + I_y q\hat{j} + I_z r\hat{k}$	(1.6)

Consider the missile to be a rigid body, and the axes chosen to be principle axes. The force and moment component equations of motion are:

$$F_x = m(\dot{u} - rv + qw) \tag{1.7}$$

$$F_y = m(\dot{v} - pw + ru) \tag{1.8}$$

$$F_z = m(\dot{w} - qu + pv) \tag{1.9}$$

$$M_x = I_x\dot{p} \tag{1.10}$$

$$M_y = I_y\dot{q} - (I_z - I_x)\,rp \tag{1.11}$$

$$M_z = I_z\dot{r} - (I_x - I_y)\,pq \tag{1.12}$$

If we consider the gravity components of force as inputs to the system which are trimmed out in steady flight by command bias signals we may write the applied forces and moments as:

$$F_x = T - QSC_{do}(M) \tag{1.13}$$

$$F_y = QSC_y(\beta, i_y, \delta, M) \tag{1.14}$$

$$F_z = QSC_N(\alpha, i_p, \delta, M) \tag{1.15}$$

$$M_x = QSdC_l(p, \alpha, \beta, \delta, i_y, i_p, M) \tag{1.16}$$

$$M_y = QSdC_m(p, \alpha, \delta, M) \tag{1.17}$$

$$M_z = QSdC_n(\beta, i_y, \delta, M) \tag{1.18}$$

where:

T	$\equiv$	Rocket thrust, aligned with x axis	(1.19)
Q	$\equiv$	Dynamic pressure $= \frac{1}{2}\rho V^2$	(1.20)
M	$\equiv$	Mach number	(1.21)
S	$\equiv$	Reference area, (usually body cross section)	(1.22)
C_{do}	$\equiv$	Nondimensional drag coefficient	(1.23)
C_y	$\equiv$	Nondimensional yaw force coefficient	(1.24)

C_N	$\equiv$	Nondimensional pitch force coefficient	(1.25)
C_l	$\equiv$	Nondimensional rolling moment coefficient	(1.26)
C_m	$\equiv$	Nondimensional pitch moment coefficient	(1.27)
C_n	$\equiv$	Nondimensional rolling moment coefficient	(1.28)
d	$\equiv$	Reference moment arm (usually missile body diameter)	(1.29)
β	$\equiv$	Yaw angle of attack $\equiv$ arc sin $\frac{v}{V}$	(1.30)
α	$\equiv$	Pitch angle of attack $\equiv$ arc sin $\frac{w}{V}$	(1.31)
i_y	$\equiv$	γ $\equiv$ Control surface deflection to produce yaw forces and moments	(1.32)
i_p	$\equiv$	Control surface deflection to produce pitch forces and moments	(1.33)
δ	$\equiv$	Control surface deflection to produce rolling moment	(1.34)

From Figure 1.2 we see that γ , i_p and δ can be related to the actual control surface deflections $\delta_{Ⓝ}$ by

$$\gamma = \delta_{①} + \delta_{③} \tag{1.35}$$

$$i_p = \delta_{②} + \delta_{④} \tag{1.36}$$

$$\delta = \delta_{①} - \delta_{②} - \delta_{③} + \delta_{④} \tag{1.37}$$

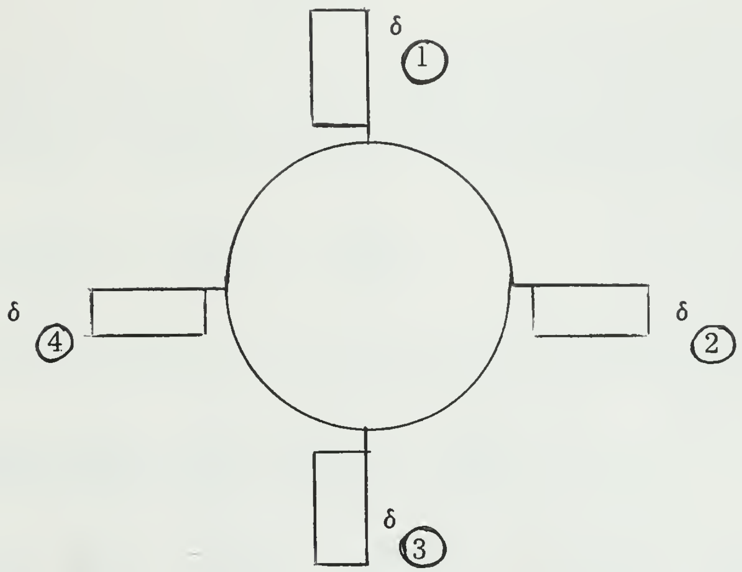

Fig. 1.2 Positive Deflection of Control Surfaces

Now consider the missile to be in the following steady state condition:

1. Constant speed (constant Mach) where

$$V = V_o = \sqrt{u_o^2 + v_o^2 + w_o^2} = \sqrt{u_o^2 + w_o^2} \tag{1.38}$$

2. Missile trimmed to constant pitch angle of attack, α_o. This means that

$$M_y = QSdC_m = 0 \tag{1.39}$$

and

$$F_z = QSC_N = \text{constant} \tag{1.40}$$

The above may be expressed by:

$$v_o = p_o = r_o = \beta_o = \delta_o = \gamma_o = 0 \tag{1.41}$$

and

$$i_p = \text{constant} \tag{1.42}$$

$$q_o = \text{constant} \tag{1.43}$$

Allow motions of v, r and p as small perturbations about the steady state values noted above, and consider their products to be negligible. We find, on substituting into the equations in which v, r and p occur:

$$F_y = QS \left(\frac{\partial C_y}{\partial \beta} \beta + \frac{\partial C_y}{\partial \gamma} \gamma + \frac{\partial C_y}{\partial \delta} \delta \right.$$

$$= m \left[V_o (\cos \beta) \dot{\beta} - V_o (\sin \alpha_o) p + V_o (\cos \alpha_o) r \right] \tag{1.44}$$

$$M_x = QSd \left(\frac{\partial C_l}{\partial p} p + \frac{\partial C_l}{\partial \beta} \beta + \frac{\partial C_l}{\partial \delta} \delta + \frac{\partial C_l}{\partial \gamma} \gamma \right) = I_x \dot{p} \tag{1.45}$$

$$M_y = QSd \left(\frac{\partial C_o}{\partial \beta} \beta + \frac{\partial C_n}{\partial \gamma} \gamma + \frac{\partial C_n}{\partial \delta} \delta \right) = I_z \dot{r} - (I_x - I_y) p q_o \tag{1.46}$$

The partial derivatives above are stability derivatives[1.1] and are obtained empirically for airframe and conditions specified by wind tunnel tests.

Substitute the below assumptions and definitions into equations (1.44) - (1.46) to arrive at the final linearized aerodynamic performance equations of the missile, equations (1.64) - (1.66).

$$\cos \beta = 1, \text{ since } \beta \text{ less than } 15^o \tag{1.47}$$

Q = constant since V is constant and ρ is constant at steady altitude (1.48)

m, I_x and I_z are constants since propellant burning is slow process (1.49)

$$p \equiv \dot{\phi} \equiv \text{roll rate} \tag{1.50}$$

$$q \equiv \dot{\theta} \equiv \text{pitch rate} \tag{1.51}$$

$$r \equiv \dot{\psi} \equiv \text{yaw rate} \tag{1.52}$$

$$A \equiv \frac{QS\ 57.3}{mV_o} \frac{\partial C_y}{\partial \beta} \tag{1.53}$$

$$B \equiv \frac{QS\ 57.3}{mV_o} \frac{\partial C_y}{\partial \gamma} \tag{1.54}$$

$$C \equiv \frac{QSd\ 57.3}{I_z} \frac{\partial C_n}{\partial \beta} \tag{1.55}$$

$$E \equiv \frac{QSd\ 57.3}{I_z} \frac{\partial C_n}{\partial \gamma} \tag{1.56}$$

$$F \equiv \frac{QSd\ 57.3}{I_x} \frac{\partial C_l}{\partial p} \tag{1.57}$$

$$G \equiv \frac{QSd\ 57.3}{I_x} \frac{\partial C_l}{\partial \delta} \tag{1.58}$$

$$H \equiv \frac{QSd\ 57.3}{I_x} \frac{\partial C_l}{\partial \beta} \tag{1.59}$$

$$L \equiv \frac{QSd\ 57.3}{I_x} \frac{\partial C_l}{\partial \gamma} \tag{1.60}$$

$$M \equiv \frac{QS\ 57.3}{mV_o} \frac{\partial C_y}{\partial \delta} \tag{1.61}$$

$$N \equiv \frac{QSd\ 57.3}{I_z} \frac{\partial C_n}{\partial \delta} \tag{1.62}$$

Table A.1, Appendix A, tabulates the above aerodynamic coefficients at three pitch angles:

$$\alpha_o = 0^o, 12^o, 24^o \tag{1.63}$$

Yaw force equation:

$$(A\beta + B\gamma + M\delta) = \beta - \dot{\phi} \sin \alpha_o + \dot{\psi} \cos \alpha_o \tag{1.64}$$

Yaw moment equation:

$$C\beta + E\gamma + N\delta = \ddot{\psi} - \cancel{\left(\frac{I_x - I_y}{I_z}\right) \dot{\phi}\, \dot{\theta}_o} \quad \text{Negligible term} \tag{1.65}$$

Roll moment equation:

$$G\delta + F\phi + H\beta + L\gamma = \ddot{\phi} \tag{1.66}$$

The above development is based on information received from Convair (Pomona) and roughly parallels a more rigorous development of linearized aircraft performance equations outlined in Reference 1.1, Sections II - I through II - 8.

Having the aerodynamic performance equations we need now to consider the control systems in yaw and roll (pitch being considered constant in this analysis).

The following sensors are used in the missile:

1. Accelerometers which in effect measure $\frac{F_z}{m}$ and $\frac{F_y}{m}$, or with suitable scale factors, $(A\beta + B\gamma + M\delta)$ and $(A\alpha + B\gamma + M\delta)$.

2. Rate gyros which measure $\dot{\phi}$, $\dot{\psi}$, $\dot{\theta}$.

The accelerometers provide information for feedback comparison with command signals and the angular rate signals are used for damping.

The sensor outputs used in this analysis are related to the sensed quantities as shown below:

$$(A\beta + B\gamma + M\delta)'(s) = G_1(s)(A\beta + B\gamma + M\delta)(s) \tag{1.67}$$

$$\dot{\phi}'(s) = G_2(s)\, \dot{\phi}(s) \tag{1.68}$$

$$\dot{\psi}'(s) = G_3(s)\, \dot{\psi}(s) \tag{1.69}$$

where the primed quantities are the sensor outputs and the sensors are quadratic of the form

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1.70}$$

The control system equations for the autopilots considered in this analysis are the yaw control equation

$$\dot{\gamma}_c(s) = [K_1 (A\beta + B\gamma + M\delta)'(s) + (K_2 + K_3 s)\dot{\psi}'(s)] \tag{1.71}$$

and the roll control equation

$$\dot{\delta}_c(s) = -[\frac{K_4}{s} + K_5 + K_6 s]\dot{\phi}'(s) \tag{1.72}$$

These are integrated through rate servos to control surface deflections

$$\gamma(s) = \frac{G_4(s)}{s}\dot{\gamma}_c(s) \tag{1.73}$$

and

$$\delta(s) = \frac{G_5}{s}\dot{\delta}_c(s) \tag{1.74}$$

where G_4 and G_5 are of the form indicated in equation (1.70).

A functional diagram representation of the system is shown in Figure 1.3.

1.3 Performance Specifications

Consideration of the equations representing the aerodynamic response functions of Figure 1.3 shows that interaction exists between $(A\beta + B\gamma + M\delta)$, $\dot{\phi}$, $\dot{\psi}$ and γ and δ at $\alpha_o > 0^o$.

Since the system can be decoupled at $\alpha_o = 0^o$, specifications for response at that pitch angle are to be met first. As set forth by

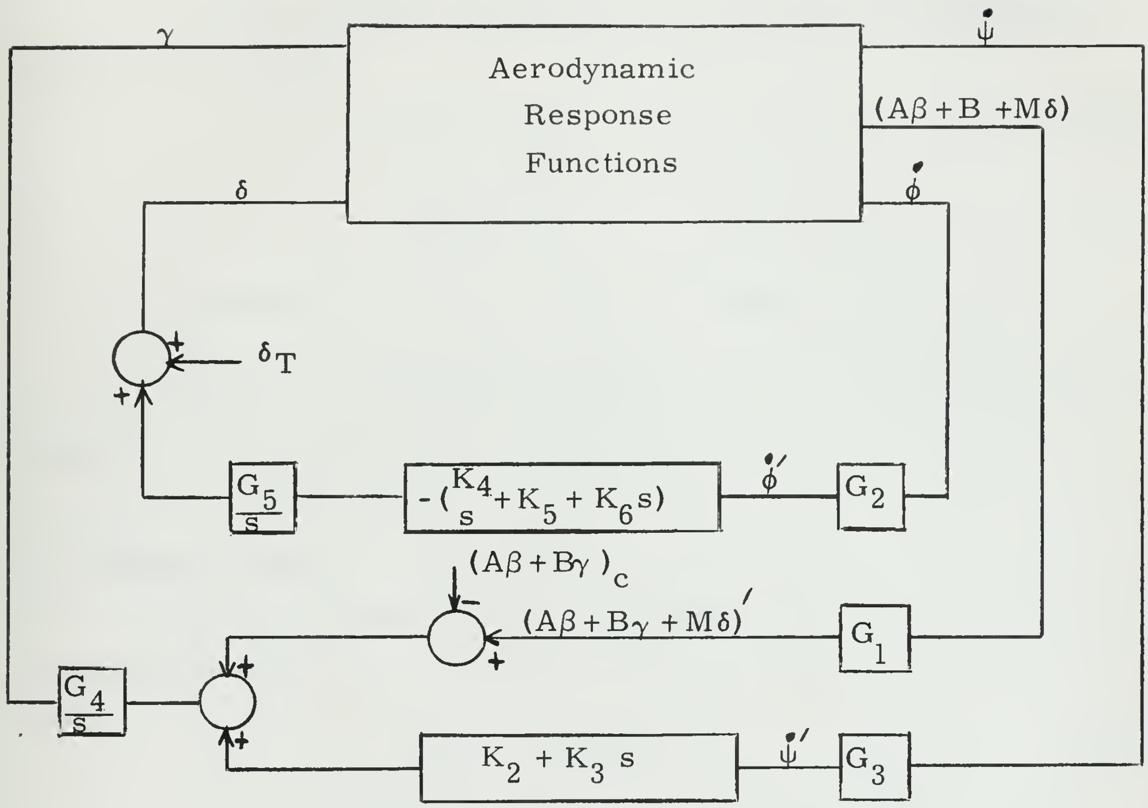

Fig. 1.3 Missile System Functional Diagram

Convair in stating the problem, the responses to be achieved (in uncoupled mode) are:

1. Time constant of the yaw subsystem to step command $(A\beta + B\gamma)_c$ to be less than .6 seconds. (Time constant here is defined as the time for the quantity to reach 63% of its steady state value.)

2. Time constant of the roll subsystem response to a step torque (δ_T) to be less than .5 second. (The time for ϕ to decay to less than 37% of its peak value).

The only requirement in coupled modes ($\alpha_o = 12^o, 24^o$) is that the system be stable and reasonably damped in response to step torque, δ_T, or step command, $(A\beta + B\gamma)_c$.

In addition, other considerations have placed the following limits on K_3 and K_6:

$$K_3 \leq .40 \tag{1.75}$$

$$K_6 \leq .021 \tag{1.76}$$

All gains ($K_1 \ldots\ldots K_6$) are to be considered positive.

Thus we have the system, and the performance specifications to be met. The remainder of this chapter will describe briefly the steps to be followed in the system analysis.

1.4 Idealized System

Consider a system in which the servos and sensors have unity transfer functions:

$$G_1 \ldots\ldots G_5 = 1 \tag{1.77}$$

The missile with "perfect" sensors and servos will hereafter be called the idealized system. The idealized system will be considered first, and then the effects of non-ideal servos and sensors will be discussed. It is sufficient to say that the idealized system is a reasonable approximation to the actual system if it can be shown that the servos and sensors introduce into the actual system additional modes of low amplitude which are far removed in frequency from the natural modes of the idealized system. This matter will be discussed in Chapter 6. For the idealized system we may consider the aerodynamic and control equations in the following form:

$$(A\beta + B\gamma + M\delta) = \dot{\beta} - \dot{\phi} \sin \alpha_o + \dot{\psi} \cos \alpha_o \tag{1.78}$$

$$C\beta + E\gamma + N\delta = \ddot{\psi} \tag{1.79}$$

$$GS + F\phi' + H\beta + L\gamma = \ddot{\phi} \tag{1.80}$$

$$\gamma = \frac{1}{s}[\, K_1 (A\beta + B\gamma + M\delta) + K_2 \dot{\psi} + K_3 \ddot{\psi}] \tag{1.81}$$

$$\delta = -\frac{1}{s}\left[K_4\phi + K_5\dot{\phi} + K_6\ddot{\phi} \right] \tag{1.82}$$

1.5 Procedure of Investigation

The investigations of subsequent chapters will

1. Discuss analytic methods of selecting gains to provide specified performance.

2. Use analogue simulation (guided by the methods discussed above) to select a suitable set of gains.

3. Discuss the possibility of compensating networks within the missile to eliminate or attenuate the effects of coupling on desired response.

CHAPTER 2

ANALYTICAL STUDY OF THE COUPLED SYSTEM

2.1 General

In Section 1.4 of Chapter 1 are shown the linearized differential equations of the idealized system. These can be put into the form of a "system determinant" by gathering all of the variables in each equation to one side, the other side being zero. If this be done, and LaPlace transform notation be adopted, there results the system determinant:

$$\begin{array}{c} \\ \Delta = \end{array} \begin{array}{c} \begin{array}{ccccc} \beta & \dot{\psi} & \dot{\phi} & \gamma & \delta \end{array} \\ \begin{vmatrix} (s-A) & \cos\alpha_o & -\sin\alpha_o & -B & -M \\ -C & s & 0 & -E & -N \\ -H & 0 & (s-F) & -L & -G \\ 0 & 0 & \frac{X}{s^2} & 0 & 1 \\ -K_1A & -Y & 0 & (s-K_1B) & -K_1M \end{vmatrix} \end{array} \qquad (2.1)$$

where

$$X \equiv (K_6 s^2 + K_5 s + K_4)$$

$$Y \equiv (K_3 s + K_2)$$

Expansion of this determinant yields the system characteristic equation. This is a sixth order polynomial in s in whose coefficients appear a considerably involved mixture of the controller gains. Appendix B gives this characteristic equation where the aerodynamic coefficients for $\alpha_o = 24^o$ have been entered.

The objectives of the analytical investigation are:

1. To determine how to fix upon a set of gains K_1 through

K_6 which produce a stable system and

2. To determine a set of gains which will provide for meeting the specified preformance at $\alpha_o = 0$ described in Chapter 1, Section 1.3 and

3. To select these gains not only to meet objectives 1. and 2., but further to obtain conditions in which the disturbance in roll caused by insertion of yaw command and the disturbance in yaw casued by roll torque disturbances are acceptably low and of reasonable damping.

Inspection of the characteristic equation reveals that several inequalities can be obtained from the expressions for the coefficients of powers of s which must be satisfied to prevent occurence of roots of the characteristic equation in the right half of the s plane. Given a set of gains, the test for presence of sign change in the equation can be readily performed.

It would also be possible to apply the Routh Criterion, at the expense of additional labor, for a given set of gains, to obtain further information as to the nature of the roots.

These, and other procedures that might be derived afford little in the way of a guide to selection of a new set of gains to test when one set has resulted in an unstable condition.

There is the possibility of forming from (2.1) a set of non-interacting transfer functions from which a block diagram of the system could be constructed. With manipulation of this diagram and the use of root locus methods, the roots for a given set of gains could be found. Unfortunately there would be at least three inner loops of such a diagram whose roots are gain dependent, and a very considerable amount of tedious and repetitious graphical work would be involved, even if each new attempt could be based on a set of gains choice of which was guided by the last trial.

It is apparent that it would be desirable to have a more orderly approach in which less futile graphic or algebraic work is expended in arriving at the conclusion that the last guess was unsatisfactory before making a new, more or less intelligent, guess.

The object of the following section is to discuss a procedure which, while not avoiding trial and error and not avoiding tedium, at least puts to good use the experience gained by past results.

This procedure begins with manipulations of the characteristic equations of the uncoupled roll and yaw subsystems. These equations are of low order and quite straight forward so that the effect of gain changes on the resultant responses are readily seen.

2.2 An Algebraic Approach to a Stable System Characteristic Equation

In Chapter 3 is derived a block diagram interpretation of the system equations which results in a new form of the system determinant where in the coupling effects and the subsystems are clearly depicted.

When (3.7) of Chapter 3 is reformed for $\alpha_o = 0$ we have

$$\begin{array}{c} \underline{\text{Yaw}}\ \underline{\text{Subsystem}} \\ \Delta = \left|\begin{array}{ccc|cc} 1 & -F & -F_{12} & 0 & 0 \\ -F_6 & 1 & -F_5 & 0 & 0 \\ -F_3 & -F_1 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & -F_{10} \\ 0 & 0 & 0 & -F_{11} & 1 \end{array}\right| \\ \underline{\text{Roll}}\ \underline{\text{Subsystem}} \end{array} \tag{2.2}$$

Expansion of (2.2) will reveal a result which is the product of the results obtained by expanding those sections of (2.2) labelled Yaw Subsystem and Roll Subsystem as if they were themselves determinants.

When the block diagram Fig. 3.7 of Chapter 3 is considered at $\alpha_o = 0$ that diagram divides into two new diagrams of the roll and yaw subsystems. Using the methods described in Chapter 3, one can obtain the subsystem determinants which are the subdeterminants

within (2.2).

Then if we let

$$\Delta_r \equiv \begin{vmatrix} 1 & -F_{10} \\ -F_{11} & 1 \end{vmatrix} = \frac{s^2 (s-F) + G(K_6 s^2 + K_5 s + K_4)}{s^2 (s-F)} \qquad (2.3)$$

and

$$\Delta_y \equiv \begin{vmatrix} 1 & -F_{13} & -F_{12} \\ -F_6 & 1 & -F_5 \\ -F_3 & -F_1 & 1 \end{vmatrix} =$$

$$\Big[\frac{s^3 + (-K_1 B - A - EK_3) s^2 + (AK_1 B - C - EK_2 + EAK_3 - BCK_3 - BK_1 A) s}{s(s-K_1 B)(s-A)} +$$

$$\frac{(CK_1 B = EAK_2 + EAK_1 - BCK_2)}{s(s-K_1 B)(s-A)} \Big] \qquad (2.4)$$

Employing the same analog with Cramer's Rule as is used in Chapter 3 we can express the outputs due $(A\beta + B\gamma)_c$ and δ_T in this $\alpha_o = 0$ case as

$$\frac{\dot{\phi}(s)}{\delta_T(s)} = \frac{\begin{vmatrix} 0 & -F_{10} \\ 1 & 1 \end{vmatrix}}{\Delta_r} \qquad (2.5)$$

$$\frac{(A\beta + B\gamma(s)}{(A\beta + B\gamma(s)} = \frac{AF_A \begin{vmatrix} 1 & -F_{13} & 1 \\ -F_6 & 1 & 0 \\ -F_3 & -F_1 & 0 \end{vmatrix}}{\Delta_y} + \frac{BF_A \begin{vmatrix} 1 & -F_{13} & -F_{12} \\ 0 & 1 & -F_5 \\ 0 & -F_1 & 1 \end{vmatrix}}{\Delta_y} \qquad (2.6)$$

When (2.5), (2.6) are expanded, substitutions made for the defined transfer functions, and simplification made, applying a step input $(A\beta + B\gamma)_c(s) = \frac{P}{s}$ and $\delta_T(s) = \frac{R}{s}$.

$$\phi(s) = \frac{RG}{(S^3 + (GK_6 - F)S^2 + K_5 S + K_4)} \tag{2.7}$$

$$(A\beta \ + B\gamma)(s) = \frac{-K_1 \, PA \, [\, Bs - E]}{S\,[\, S^3 + (-K_1 B - A - EK_3)\, S^2 + \ldots\ldots]}$$

$$+ \frac{-K_1 \, PB \, [\, S\,(S-A) + C]}{S[\, S^3 + (-K_1 B - A - EK_3) S^2 + \ldots\ldots]} \tag{2.8}$$

In the results of (2.7) and (2.8) it is clear that the modes which exist are completely specified by the denominators and that in each case these are 3 in number. It is further apparent that the zeros of both closed loop functions are due to aerodynamic coefficients.

Chu[2.1] has derived a large number of forms of "desired" characteristic equations on the assumption that the performance is dominated by a pair of complex roots and has prepared graphical aids to the rapid determination of the pole-zero configuration of the closed loop function which reflects a specified time-domain response to a step input. With these it is possible to rapidly obtain a set of gains for the separate subsystems to achieve a specified response. However, there is no certainty that the assumption of "2nd order dominance" in the subsystem will produce a set of gains which will make the coupled system stable.

It is possible, though, to assume a set of gains and rapidly determine the residue coefficients of the modes that exist using methods described by Thaler.[2.2] By this method a check can be quickly obtained upon the ability of a stated set of gains to meet the specifications at $\alpha_o = 0$. A map can then be prepared of the locations of roots of the characteristic equations of the subsystems which do satisfy the $\alpha_o = 0$ specifications.

Having obtained a starting set of values of the gains, these can be entered into the coupled system characteristic equation (Appendix B) and that equation factored, the roots plotted with the roots of the

characteristic equations of the uncoupled subsystems, and the trend noted.

A second trial for the roots can be guided by the concept that increased stability of the coupled system will be accompanied by increased "sluggishness" in the $\alpha_o = 0$ modes of the subsystems. In the actual carrying out of this process it is also noted that transient oscillating frequencies of $\alpha_o = 0$ modes tend higher from the first choice of subsystem root locations. By following a path on which the complex pair of roots of each subsystem increase in natural frequency but remain at constant damping ratio the real root moves toward the origin, and convergence to stability of the coupled system characteristic equation is apparently more rapid than on other paths.

Thus there does exist one orderly, albeit tedious, process for converging upon a set of gains which produces stability of the coupled system and yet allows the subsystems to meet the $\alpha_o = 0$ specifications.

"Bold spotting" allowed convergence to a stable set of gains in three tries using these concepts.

Somewhat less tedious labor is involved if one does not observe the constraint of specifications too rigorously, merely working with the subsystem characteristic equations to produce stability in the coupled system characteristic equation without too much regard for specifications, then trimming to achieve the specifications at $\alpha_o = 0$.

Such a procedure is, then, capable of satisfying objective (1) of Section 2.1. It is quite apparent that repetitive factoring of the coupled system characteristic equation as would be necessary with this (and most other) methods to reach the other objectives is simply beyond reason without automatic computing facilities.

The conclusion is therefore drawn that this method does set forth a concept which is useful in guiding one to a stable set of gains for an analogue simulation of the problem, but is otherwise of academic interest, and totally devoid of practicality in achieving the other objectives of Section 2.1.

For this reason all work in "trimming up" the system was done by

analogue simulation as discussed in Chapter 4.

2.3 Steady State Analysis of the System

In the preceding section it was concluded that setting the gains in the system to meet all the stated objectives is not a practical task for analytical study, that rather it is an undertaking more appropriate to the analogue computer. Considerable useful information for a computer study can be gained by analyzing the features of the variables considered as outputs in the steady state.

The methods described in Chapter 3 for solving the equations will be used here. The uncoupled mode at $\alpha_o = 0$ will be dealt with first.

1. Uncoupled Roll Subsystem

In Section 2.2 it was shown that at $\alpha_o = 0$ the roll system charactoristic equation is given by

$$\Delta_r = \begin{vmatrix} 1 & -F_{10} \\ -F_{11} & 1 \end{vmatrix} = \frac{s^2(s-F) + G(K_6 s^2 + K_5 s + K_4)}{s^2(s-F)} \tag{2.3}$$

It was also shown that

$$\frac{\dot{\phi}}{\delta_T}(s) = \frac{\begin{vmatrix} 0 & -F_{10} \\ 1 & 1 \end{vmatrix}}{\Delta_r} = \frac{F_{10}}{\Delta_r} \tag{2.5}$$

Since $\frac{F_{10}}{\Delta_r}$ is analytic in the RHP, the final value theorem is applicable. For $\delta_T(s) = \frac{R}{s}$

$$\lim_{t \to \infty} \phi(t) = \lim_{s \to 0} [s\phi(s)] = \frac{s\left[\frac{R}{s} \frac{G}{s(s-F)}\right]}{\dfrac{s^3 + (GK_6 - F)s^2 + GK_5 s + GK_4}{s^2(s-F)}} = 0 \tag{2.9}$$

Thus the roll system ultimately recovers from a roll step disturbance.

2. Uncoupled Yaw Subsystem

From (2.4) the characteristic equation of the uncoupled yaw subsystem is

$$\Delta_y = \frac{[\, s^3 + (-K_1B - A - EK_3)s^2 + (AK_1B - EK_2 + EAK_3 - BCK_3 - BK_1A)s}{s\,(s - K_1\,B)(s - A)}$$

$$+ \frac{(CK_1\,B - EAK_2 + EAK_1 - BCK_2)]}{s\,(s - K_1B)(s - A)} \qquad (2.4)$$

In (2.8) for a step input $(A\beta + B\gamma)_c(s) = \frac{P}{s}$

$$(A\beta + B\gamma)(s) = \frac{-\,PAK_1\,[\,Bs - E]}{s\,[\,s^3 + \ldots\ldots (BC - AE)(K_1 + K_2)]}$$

$$+ \frac{-PBK_1\,[\,s^2 - As + C]}{s\,[\,s^3 + \ldots\ldots (BC - AE)(K_1 + K_2)]} \qquad (2.5)$$

Then since for a stable choice of gains this function is analytic in the RHP the Final Value Theorem applies and

$$\lim_{t \to \infty} (A\beta + B\gamma)(t) = \lim_{s \to 0}\,[\,s\,(A\beta + B\gamma)(s)]$$

$$= \frac{-PK_1\,(BC - AE)}{K_1 + K_2\,(BC - AE)} = -P\left(\frac{K_1}{K_1 + K_2}\right) \qquad (2.11)$$

Applying the Cramer's Rule method of solution to the yaw subdeterminant in (2.2)

$$\frac{\dot{\psi}}{(A\beta + B\gamma)_c}(s) = \frac{-F_a\,(-F_6 - F_3F_5)}{\Delta_y} = \frac{[\,Es\,(s-A) + BC]\;\dfrac{-K_1}{s(s-A)(s-K_1B)}}{\Delta_y} \qquad (2.12)$$

or

$$\frac{\dot{\psi}}{(A\beta + B\gamma)_c}(s) = \frac{-K_1 \,[\, Es + (BC - AE)]}{[\, s^3 + \ldots\ldots (BC - AE)(K_1 + K_2)]} \qquad (2.12a)$$

Using $(A\beta + B\gamma)_c(s) = \frac{P}{s}$ again, and applying the Final Value Theorem

$$\lim_{t \to \infty} \dot{\psi}(t) = \lim_{s \to 0} s\,[\dot{\psi}(s)] = \frac{-K_1 P}{K_1 + K_2} \qquad (2.13)$$

Thus it is apparent that both $(A\beta + B\gamma)(t)$ and $\dot{\psi}(t)$ come to a constant non zero value in steady state and that $\psi(t)$ increases without limit . Their senses and magnitudes are easily calculated for comparison with computer results.

3. Coupled System

In performing the steady state analysis at $\alpha_o > 0$ it is convenient to let $s \to 0$ in the system determinant (3.7) at the outset since that determinant appears in the denominator of the Cramer's Rule solution for each variable of interest. In the following statements, the fact that this has been done will be signified by $[\Delta]_{ss}$ where

$$\lim_{s \to 0} \Delta = \frac{1}{s^3} \begin{vmatrix} 1 & \frac{K_2}{K_1 B} & \frac{A}{B} & 0 & \frac{M}{B} \\ -E & 0 & -C & 0 & -N \\ \frac{B}{A} & \frac{\cos \alpha_o}{A} & 1 & \frac{\sin \alpha_o}{A} & \frac{M}{A} \\ \frac{L}{F} & 0 & \frac{H}{F} & 1 & \frac{G}{F} \\ 0 & 0 & 0 & K_4 & 0 \end{vmatrix} = \frac{1}{s^3}\,[\,\Delta\,]_{ss} \qquad (2.14)$$

Other notation is as explained in Chapter 3. It is assumed throughout that a stable situation exists so that the Final Value Theorem is applicable.

a. ϕ due to roll disturbances $\delta_T(s) = \frac{R}{s}$

$$\phi(s) = \frac{\frac{1}{s}\Delta_{rr}\frac{R}{s}}{\Delta} \tag{2.15}$$

$$\lim_{t\to\infty}\phi(t) = \lim_{s\to 0} s\,[\,\phi(s)] = \frac{s\frac{R}{s^2}[\,\Delta_{rr}]_{ss}}{\frac{1}{s^3}[\,\Delta\,]_{ss}} = \frac{R\,s^2\,[\,\Delta_{rr}]_{ss}}{[\,\Delta\,]_{ss}} \tag{2.15a}$$

Evaluation of the determinants shows $\phi(\infty) = 0$ when a roll disturbance is applied.

b. ϕ due to yaw command $(A\beta + B\gamma)_c(s) = \frac{P}{s}$

$$\phi(s) = \frac{\frac{1}{s}\Delta_{yr}\frac{P}{s}}{\Delta} \tag{2.16}$$

$$\lim_{t\to\infty}\phi(t) = \lim_{s\to 0} s\,[\,\phi(s)] = \frac{s\frac{P}{s^2}[\,\Delta_{yr}]_{ss}}{\frac{1}{s^3}[\,\Delta\,]_{ss}} \tag{2.16a}$$

Again evaluation of the determinants shows $\phi(\infty) = 0$ when a yaw command is applied.

c. $\dot{\psi}$ due to a yaw command $(A\beta + B\gamma)_c(s) = \frac{P}{s}$

$$\psi(s) = \frac{\Delta_{yy}\frac{P}{s}}{\Delta} \tag{2.17}$$

$$\lim_{t\to\infty}\dot{\psi}(t) = \lim_{s\to 0} s\,[\,\dot{\psi}(s)] = \frac{P\,s^3\,[\,\Delta_{yy}]_{ss}}{[\,\Delta\,]_{ss}} \tag{2.17a}$$

Evaluation of the determinants shows that $\dot{\psi}$ goes to the constant

$$[\dot{\psi}(t)]_{ss} = \frac{-K_1 P}{K_1 \cos \alpha_o + K_2} \tag{2.17b}$$

and that ψ increases without limit as $t \rightarrow \infty$

d. $\dot{\psi}$ due to roll disturbance $\delta_T(s) = \frac{R}{s}$

$$\dot{\psi}(s) = \frac{\frac{R}{s} \Delta_{ry}}{\Delta} \tag{2.18}$$

$$\lim_{t \rightarrow \infty} \dot{\psi}(t) = \lim_{s \rightarrow 0} s[\dot{\psi}(s)] = \frac{R\, s^3 [\Delta_{ry}]_{ss}}{[\Delta]_{ss}} \tag{2.18a}$$

Evaluation of the determinant shows that $\dot{\psi}(\infty) = 0$.

Further evaluation of the integral shows that $\psi(\infty) = 0$.

This information on steady state values is of use in correlating the analogue computer results in Chapter 4 with the physical operation of the system.

CHAPTER 3

COMPENSATION

3.1 General Approach

The general problem of preventing interaction between controlled quantities in multiple-loop systems has been discussed by Tsien,[3.1] Truxal[3.2] and others. Among these Chu[2.1] has developed a generalized theory of multiple-loop systems including techniques for the determination of compensation necessary to prevent interaction. Chu employs methods of manipulation with the system determinant with the aid of block diagrams. The methods of Tsien and Chu are quite parallel in theory, the choice between them being a subjective matter. Chu's method has been chosen for application to this problem since that technique is more familiar to the authors.

A brief description of the basic elements of Chu's approach to mulitple-loop control systems is included in Appendix C and no further detailed explanation will be given here.

The problem of analyzing the effects of interactions in this system is essentially one of maintaining contact with the physical hardware involved. Particularly, a block diagram possesses the general appeal of being a reasonably tangible representation of the relations among the physical variables of a system, provided that diagram does not endeavor to lump too many quantities and interrelations into a single complicated block transfer function.

A diagram which, it is believed, will provide a reasonably physical view of the problem will first be derived. It employs the most basic set of transfer functions obtainable from the linearized equations.

Consider the set of linearized equations in the form of a determinant:

$$
\begin{array}{l}
\text{(I)} \\ \text{(II)} \\ \text{(III)} \\ \text{(IV)} \\ \text{(V)}
\end{array}
\begin{vmatrix}
(s-A) & \cos\alpha_o & -\sin\alpha_o & -B & -M \\
-C & s & 0 & -E & -N \\
-H & 0 & (s-F) & -L & -G \\
0 & 0 & \frac{X}{s^2} & 0 & 1 \\
-K_1 A & -Y & 0 & (s-K_1 B) & -K_1 M
\end{vmatrix}
\begin{array}{l}
= 0 \\ = 0 \\ = 0 \\ = 0 \\ = 0
\end{array}
\qquad (3.1)
$$

where (I) $\equiv$ yaw force equation

(II) $\equiv$ yaw moment equation

(III) $\equiv$ roll moment equation

(IV) $\equiv$ roll control equation

(V) $\equiv$ yaw control equation

and $X \equiv (K_6 s^2 + K_5 s + K_4)$

$Y \equiv (K_2 + K_3 s)$

From each equation represented by a row of the determinant a single variable is selected to be described as a function of all the others. For manipulative ease, a transfer function describing each relation is defined:

β	$\dot{\psi}$	$\dot{\phi}$	γ	δ
s - A	$-\cos\alpha_o$	$\sin\alpha_o$	B	M

(3.2)

Equation (I)

$$\frac{\beta}{\dot{\psi}}(s) = \frac{-\cos\alpha_o}{(s - A)} \equiv F_1(s) \qquad (3.2a)$$

$$\frac{\beta}{\dot{\phi}}(s) = \frac{\sin \alpha_o}{(s-A)} \equiv F_2(s) \tag{3.2b}$$

$$\frac{\beta}{\gamma}(s) = \frac{B}{(s-A)} \equiv F_3(s) \tag{3.2c}$$

$$\frac{\beta}{\delta}(s) = \frac{M}{(s-A)} \equiv F_4(s) \tag{3.2d}$$

The theorem of superposition suggests representation of $\beta(s)$ by the elemental block diagram.

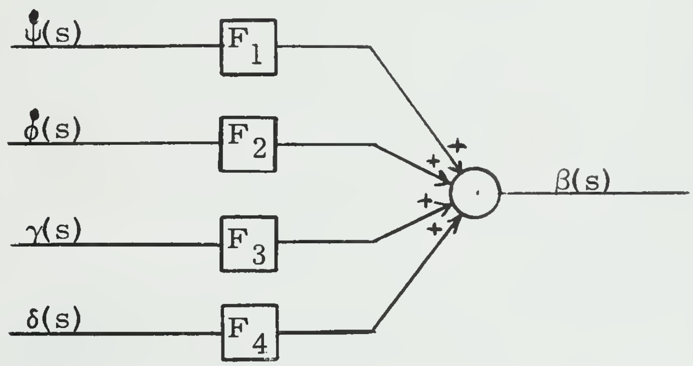

Fig. 3.1

$$\begin{array}{c||c|c|c|c} \dot{\psi} & \beta & \gamma & \delta & \dot{\phi} \\ \hline s & C & E & N & 0 \end{array} \tag{3.3}$$

Equation (II)

$$\frac{\dot{\psi}}{\beta}(s) = \frac{C}{s} \equiv F_5(s) \tag{3.3a}$$

$$\frac{\dot{\psi}}{\gamma}(s) = \frac{E}{s} \equiv F_6(s) \tag{3.3b}$$

$$\frac{\dot{\psi}}{\delta}(s) = \frac{N}{s} \equiv F_7(s) \tag{3.3c}$$

$$\frac{\dot{\psi}}{\dot{\phi}}(s) = 0 \tag{3.3d}$$

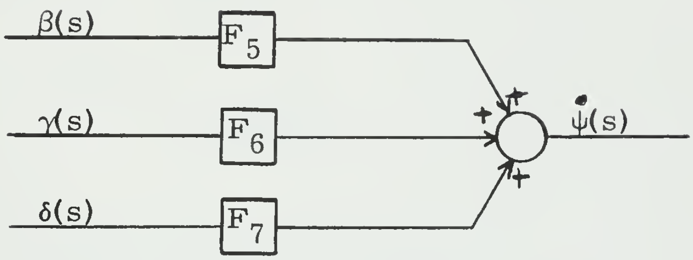

Fig. 3.2

$$\begin{array}{c||c|c|c|c} \dot{\phi} & \beta & \gamma & \dot{\psi} & \delta \\ \hline (s-F) & H & L & 0 & G \end{array} \tag{3.4}$$

Equation (III)

$$\frac{\dot{\phi}}{\beta}(s) = \frac{H}{(s-F)} \equiv F_8(s) \tag{3.4a}$$

$$\frac{\dot{\phi}}{\gamma}(s) = \frac{L}{(s-F)} \equiv F_9(s) \tag{3.4b}$$

$$\frac{\dot{\phi}}{\delta}(s) = \frac{G}{(s-F)} \equiv F_{10}(s) \tag{3.4c}$$

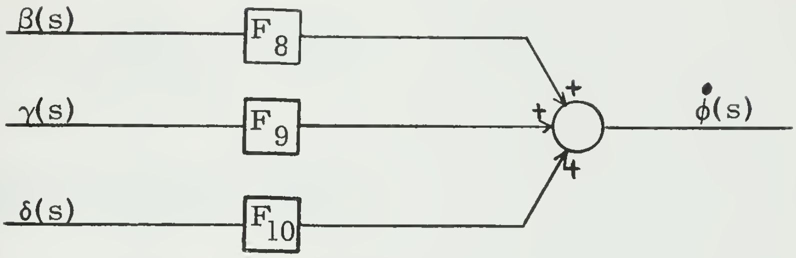

Fig. 3.3

δ	β	γ	$\dot{\psi}$	$\dot{\phi}$
1	0	0	0	$\frac{-X}{s^2}$

(3.5)

Equation (IV)

$$\frac{\delta}{\dot{\phi}}(s) = \frac{-X}{s^2} \equiv F_{11}(s) \tag{3.5a}$$

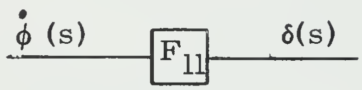

Fig. 3.4

γ	β	$\dot{\psi}$	δ	$(A\beta + B\gamma)_c$
$(s-K_1B)$	K_1A	Y	K_1M	$-K_1$

(3.6)

Equation (V)

It will be noted that provision has been made for the insertion of a yaw command signal which subtracts from the sensed side acceleration $(A\beta + B\gamma + M\delta)$ as indicated by Figure 1.3 of Chapter 1.

$$\frac{\gamma}{\beta}(s) = \frac{K_1 A}{(s-K_1 B)} \equiv F_{12}(s) \tag{3.6a}$$

$$\frac{\gamma}{\dot{\psi}}(s) = \frac{Y}{(s-K_1 B)} \equiv F_{13}(s) \tag{3.6b}$$

$$\frac{\gamma}{\delta}(s) = \frac{K_1 M}{(s-K_1 B)} \equiv F_{14}(s) \tag{3.6c}$$

$$\frac{\gamma}{(A\beta+B\gamma)_c}(s) = \frac{-K_1}{(s-K_1 B)} \equiv F_a(s) \tag{3.6d}$$

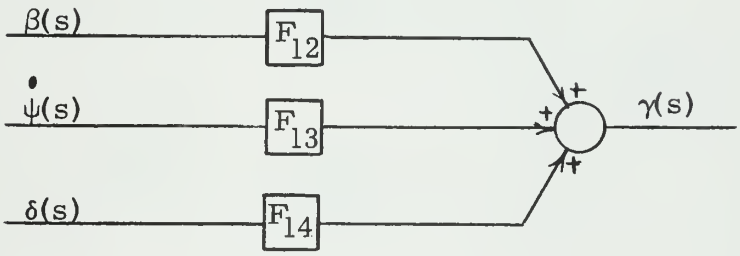

Fig. 3.5

The interrelations existing among the system variables may then be described by a block diagram as shown in Figure 3.6. Provision of a node for introduction of a roll disturbance torque δ_T will be noted.

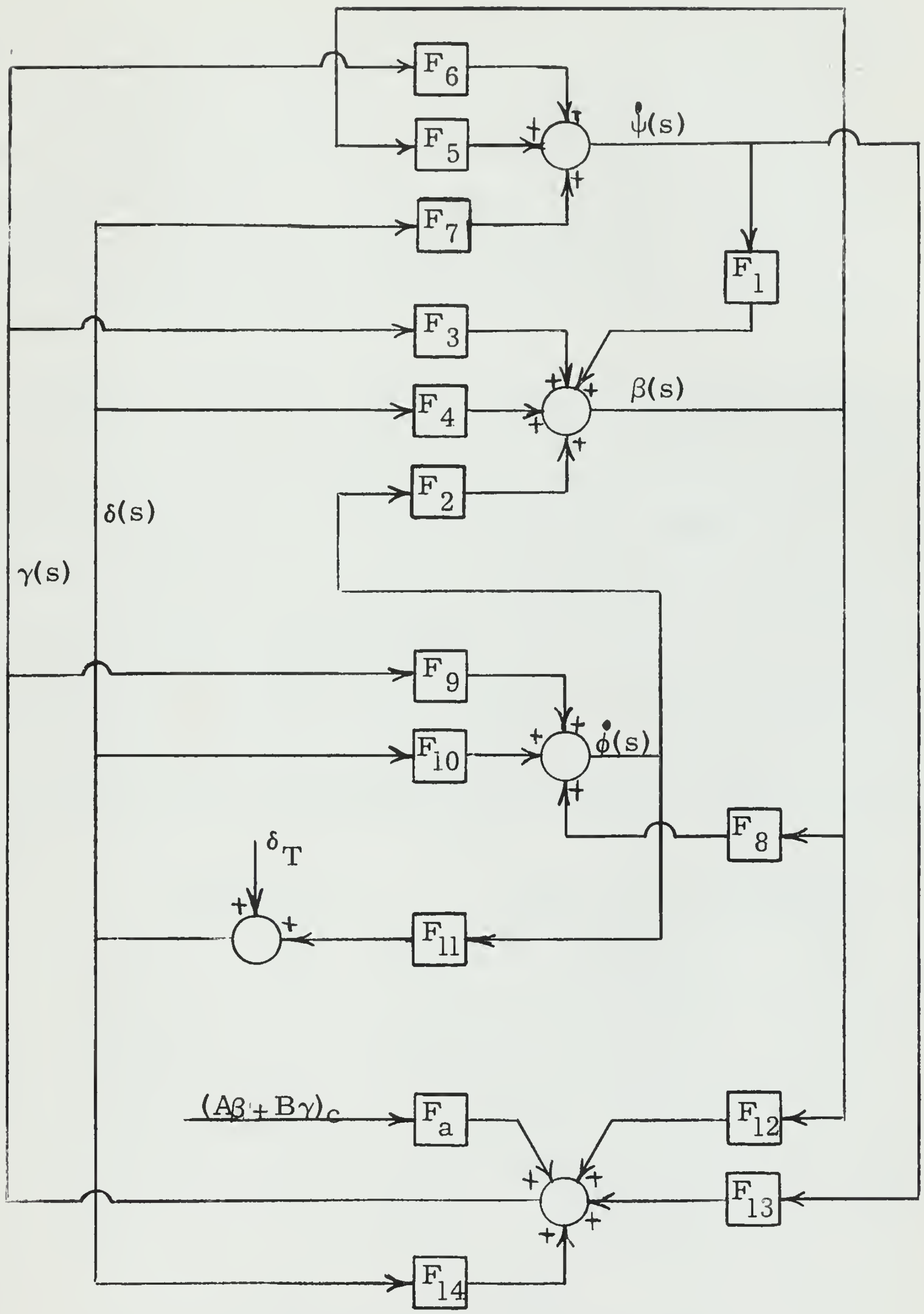

Fig. 3.6 System Block Diagram

The method of Chu requires translation of the block diagram of a system such as depicted in Figure 3.6 into a "Standardized Block Diagram". In this "standardized" diagram certain manipulations to Figure 3.6 are made to satisfy a set of conventions which allow a symmetry to exist between the "standardized" diagram and the system determinant and which allow the writing of the system determinant by inspection. These are described in Appendix C. When the necessary adjustments are made the "standardized" block diagram takes on the appearance of Figure 3.7.

In Figure 3.7 it will be observed that the manner of redrawing Figure 3.6 to produce the "standardized block diagram" has resulted in the division of the coupled system into regions which can be referred to as the roll and yaw subsystems. It is readily apparent that F_9 and F_8 represent the coupling effects of yaw into roll while F_2, F_4, F_7 and F_{14} clearly show the coupling effects of roll into yaw. Inspection of the transfer functions represented by $F_2, F_4, F_7, F_8, F_9, F_{14}$ immediately show that at $\alpha_o = 0$ the system separates into two subsystems.

By application of the rules (Appendix C) for writing the system determinant from the "standardized block diagram", that determinant becomes:

$$\begin{array}{c} \text{Yaw Subsystem} \qquad\qquad \text{Roll to Yaw Couplings} \\ \left| \begin{array}{ccc|cc} 1 & -F_{13} & -F_{12} & 0 & -F_{14} \\ -F_6 & 1 & -F_5 & 0 & -F_7 \\ -F_3 & -F_1 & 1 & -F_2 & -F_4 \\ \hline -F_9 & 0 & -F_8 & 1 & -F_{10} \\ 0 & 0 & 0 & -F_{11} & 1 \end{array} \right| \\ \text{Yaw to Roll Couplings} \qquad\qquad \text{Roll Subsystem} \end{array} \tag{3.7}$$

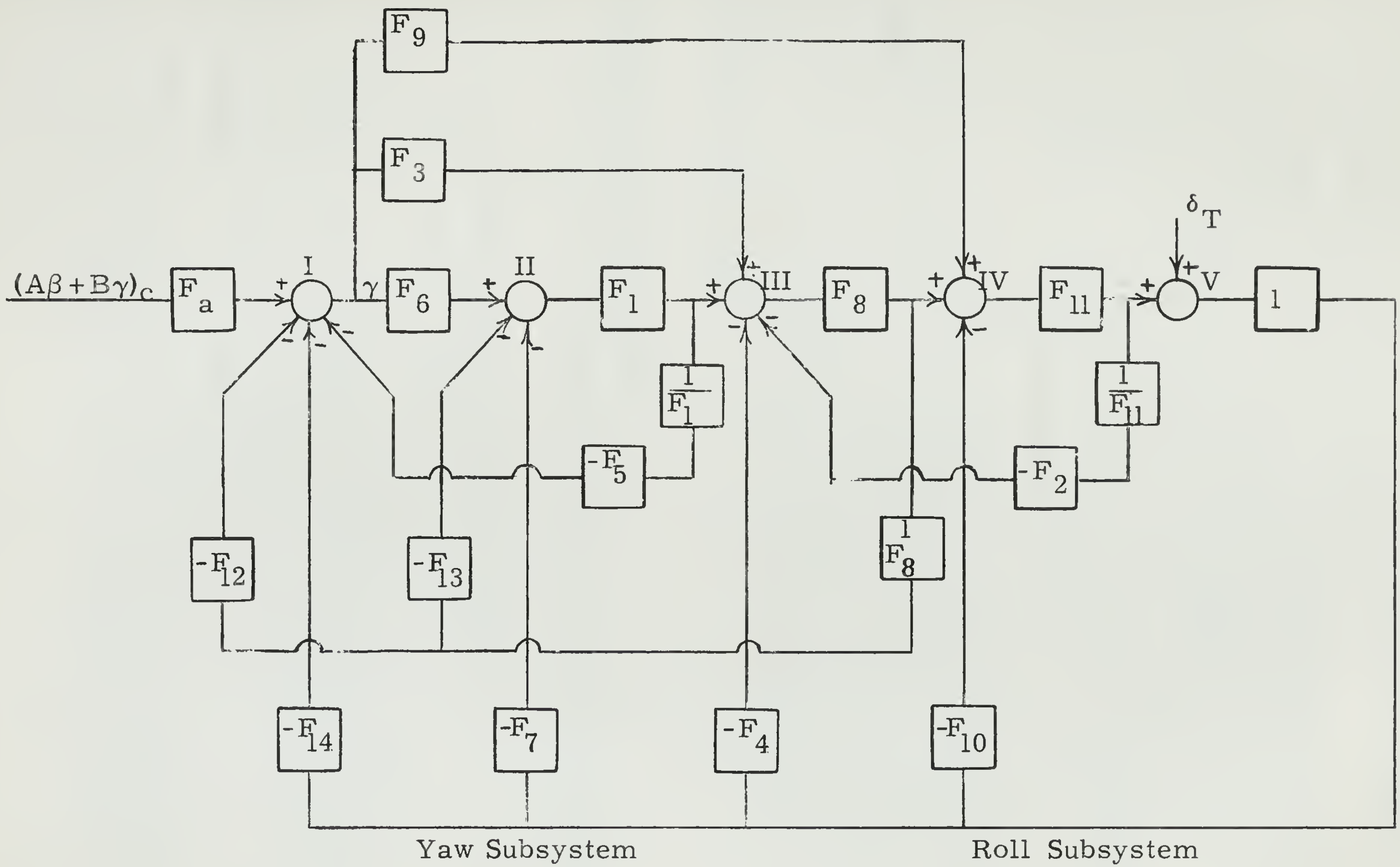

Fig. 3.7 "Standardized Block Diagram"

Equating Δ to zero and evaluating one obtains the system characteristic equation. Immediate proof that above represents the original determinant of the system can be obtained by reading each row of Δ with the appropriate substitutions.

As a consequence of the way selected for redrawing Figure 3.6 to produce Figure 3.7, Δ divides into regions illustrative of the subsystems and the intercouplings as noted above. The close parallel between this determinant and Tsien's [3.1] matrix are apparent. Also the clarity with which interactions are displayed in Figure 3.7 and in Δ would seem to some degree to refute one of the objections to the block diagram approach[3.3].

It is true that considerable effort (although simple and direct) has been put into construction of this form of Δ, but this has placed the designer in the position of having a graphic representation in Figure 3.7 of Cramer's Rule. Since each node output of Figure 3.7 represents one of the variables of interest in the problem it is merely necessary to follow Cramer's Rule to assess the output at any node due to input at any node.

3.2 Objectives and Procedures

The objectives of complete compensation will be assumed to be:

1. isolation of $\dot{\phi}$ from $(A\beta + B\gamma)_c$
2. isolation of $\dot{\psi}$ from δ_T

The introduction of any compensation will change Δ, and in this will clearly affect all outputs. This change should not produce serious effect in the desired responses. (i.e. Should not produce an unacceptable array of closed loop poles).

From the foregoing statements regarding the association between Cramer's Rule and Figure 3.7 and Δ it is clear that it is possible to solve for $\dot{\phi}(s)$ as a function of $(A\beta + B\gamma)_c$ as follows:

$$\frac{\dot{\phi}}{(A\beta + B\gamma)_c}(s) = \frac{F_a \Delta_{yr}}{\Delta} \tag{3.8}$$

Where Δ_{yr} has been formed from Δ by replacing the IV (or $\dot{\phi}$) column of Δ by a column having zeros in all row elements except the first at which unity appears (the entrance function F_a and the yaw command signal transform having been factored out of the $\dot{\phi}$ column of Δ_{yr}).

Likewise

$$\frac{\dot{\psi}}{\delta_T}(s) = \frac{\Delta_{ry}}{\Delta} \tag{3.9}$$

The desired response in yaw is given by

$$\frac{\dot{\psi}}{(A\beta + B\gamma)_c}(s) = \frac{\Delta_{yy}}{\Delta} \tag{3.10}$$

And the normal (existing, though not particularly desired) reaction of $\dot{\phi}$ to δ_T is

$$\frac{\dot{\phi}}{\delta_T}(s) = \frac{\Delta_{rr}}{\Delta} \tag{3.11}$$

Provided the missile can be prevented from yawing due to roll disturbance, no side acceleration will occur.

Then the desired goals of compensation are achieved if

1. $\Delta_{yr} = \Delta_{ry} = 0$
2. $\frac{\dot{\psi}}{(A\beta + B\gamma)_c}(s)$ is not altered by accomplishment of (1) except in the sense that Δ is changed, and that this change is not deleterious.

It is of interest to inspect both Δ_{yr} and Δ_{ry}

$$\Delta_{yr} = \begin{vmatrix} 1 & -F_{13} & -F_{12} & 1 & -F_{14} \\ -F_6 & 1 & -F_5 & 0 & -F_7 \\ -F_3 & -F_1 & 1 & 0 & -F_4 \\ -F_9 & 0 & -F_8 & 0 & -F_{10} \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \tag{3.13}$$

$$\Delta_{ry} = \begin{vmatrix} 1 & 0 & -F_{12} & 0 & -F_{14} \\ -F_6 & 0 & -F_5 & 0 & -F_7 \\ -F_3 & 0 & 1 & -F_2 & -F_4 \\ -F_9 & 0 & -F_8 & 1 & -F_{10} \\ 0 & 1 & 0 & -F_{11} & 1 \end{vmatrix} \tag{3.14}$$

As the dashed lines indicate, the expansion of Δ_{ry} and Δ_{yr} by cofactors pivots on the circled element and suppresses the indicated rows and columns. Thus

1. a compensator placed at rI, c IV will not appear in Δ_{yr}
2. a compensator placed at r V, cII will not appear in Δ_{ry}

Also of interest are Δ_{rr} and Δ_{yy}:

$$\Delta_{rr} = \begin{vmatrix} 1 & -F_{13} & -F_{12} & 0 & -F_{14} \\ -F_2 & 1 & -F_5 & 0 & -F_7 \\ -F_3 & -F_1 & 1 & 0 & -F_4 \\ -F_9 & 0 & -F_8 & 0 & -F_{10} \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix} \qquad (3.15)$$

$$\Delta_{yy} = \begin{vmatrix} 1 & 1 & -F_{12} & 0 & -F_{14} \\ -F_6 & 0 & -F_5 & 0 & -F_7 \\ -F_3 & 0 & 1 & -F_2 & -F_4 \\ -F_9 & 0 & -F_8 & 1 & -F_{10} \\ 0 & 0 & 0 & -F_{11} & 1 \end{vmatrix} \qquad (3.16)$$

From these it is noted that a compensator placed at rI, cIV does not appear in Δ_{rr} and a compensator placed at rV, cII does not appear in Δ_{yy}.

Thus one concludes that compensation at rI, cIV can possibly be used to make $\Delta_{ry} = 0$ and a compensator at rV, cII can possibly be used to make $\Delta_{yr} = 0$ and that neither compensator will appear in Δ_{yy} and Δ_{rr}.

From the way in which the system determinant is constructed from the "standardized block diagram" it is apparent that the rI, cIV element of Δ constitutes a feedback from node IV to node I in Figure 3.7. Inspection of the diagram reveals that use of this path implies that $\dot{\phi}$ is available and that modified $\dot{\phi}$ can be added algebraically to γ. Likewise rV, cII of Δ constitutes feed forward from node II to node V. $\dot{\psi}$ is physically tangible. The same question of ability to mix the modified $\dot{\psi}$ signal with δ arises here. If the compensating

networks required are realizable either exactly or approximately this question of mixing can be studied further.

Installation of compensation functions - V(s) at rV, cII and W(s) at rI, cIV in Δ, where, conveniently, no element previously existed, results in the solutions, when Δ_{yr} and Δ_{ry} are equated to zero:

$$V(s) = \frac{-Ls^2 + (LA - BH)s + (HE - LC)\cos\alpha_o}{(GE-LN)s + (-AGE-LMC - HNB + LAN + HME + GCB)} \tag{3.17}$$

$$V(s)\Big]_{\alpha_o = 24^o} = \frac{612s^2 + 143s - 146500}{-(91.2s + 42200)} \tag{3.18}$$

$$V(s)\Big]_{\alpha_o = 12^o} = \frac{133.5s^2 + 47.8s - 1548}{-[(100,300)s + 37,570]} \tag{3.19}$$

$$W(s) = \frac{\sin\alpha_o(s-A)[(CG-HMK_1)s+K_1(-BCG-LAN-HME+LCM+AGE]}{(s-K_1B)[(GE-LN)s+(-AGE-LMC-NHB+LAN+HME+GCB)]}$$

$$+\frac{s\,(s-F)[\,Ns+(CM+K_1ME-NA-BNK_1)]}{(s-K_1B)[(GE-LN)s+(-AGE-LMC-NHB+LAN+HME+GCB)]} \tag{3.20}$$

Unfortunately neither of these networks is realizable with passive circuit elements exclusively.[3.4]

Upon examination of V(s) it is found that it can be divided into

$$V(s)\Big]_{\alpha_o = 24^o} = -\frac{6.71s^2}{s+463} - \frac{1.57s}{s+463} + \frac{1600}{s+463} \tag{3.18a}$$

$$V(s)\Big]_{\alpha_o = 12^o} = -\frac{1.335\times10^{-3}s^2}{s+.376} - \frac{.478\times10^{-3}s}{s+.376} + \frac{15.5\times10^{-3}}{s+.376} \tag{3.19a}$$

An approximation using the last two terms of $V(s)\Big]_{24^o}$ is employed in the analog computer simulation with the result that while addition of this approximation to V(s) does not create an unstable system, neither does it attenuate the roll disturbance due to a yaw command input. Thus the indication is that the first term is of dominant importance.

Thus hope of a simple approximation to V(s) which would attenuate the unwanted roll response is lost. By arrangement of the analog simulator to provide an approximation to the first term of $V(s)_{24^o}$ above, about 30% decrease in the extremes of roll response to yaw command is achieved, indicating that at least partial decoupling can be achieved. Addition of the second term as above does not show any appreciable improvement over the first term alone. At this point available facilities on the computer are exhausted. The actual computer techniques employed are discussed in Chapter 4. Figure 5.7 shows the improvement achieved.

Since, if isolation is to be achieved, simulation of the first term of $V(s)\Big]_{24^o}$ is of paramount importance, it would appear that a tandem pair of derivatives on the available $\dot{\psi}$ sensor output is required. In this connection it is worthy of note that the system can be driven unstable quite easily when more gain than specified is used in the double derivative term of V(s).

The problem of introducing the modification to mix with δ has not been examined here. Available literature[3.5] does not indicate any hope of introducing the compensating signal other than to differentiate it and mix it with $\dot{\delta}_c$ rather than δ. That this procedure works has been verified on the analog computer.

It will be noted in Figures 5.8 and 5.9 that the desired responses to a yaw command, $(A\beta + B\gamma + M\delta)$ and $\dot{\psi}$, have not been adversely affected by the approximate compensation employed.

Relative to the possibility of using an expanded version of W(s) similar to that employed to approximate V(s), it will be noted

that W(s) is dependent on K_1, and thus the process of adjusting gains would involve adjustment of the compensator. This would result in a "tail-chase" operation and is not felt to be practicable. Computer facilities available do not permit examination of the effects of approximate simulation of W(s).

Since the desired approximate compensation for $\alpha_o = 12^o$ and $\alpha_o = 24^o$ are widely different, it is apparent that if the compensation for $\alpha_o = 24^o$ is installed permanently, the compensation for $\alpha_o = 12^o$ will not be proper, even in an approximate sense. Also any compensation left in permanently will have the effect of creating coupling at $\alpha_o \doteq 0$. Then some form of logical operation is necessary. It is suggested that, since the $\alpha_o = 0$ and $\alpha_o = 12^o$ responses are not greatly different (See Figures 5.1 and 5.2), the compensator for $\alpha_o = 24^o$ alone be installed and that this be switched in and out.

Inasmuch as maneuvers which would be likely to develop extreme angles of attack would probably be presaged by some violent excursion of the seeker tracking line relative to the airframe longitudinal axis, it seems conceivable that this information could be used as a basis for the necessary compensation switching. This possibility has not been examined in detail.

In conclusion, a method has been described for discussing compensation in a clear manner which involves explicit relation among the measured quantities available. A compensation which will theoretically completely isolate roll output from yaw command has been found and has been approximately verified on the analog simulator.

CHAPTER 4

COMPUTER SIMULATION

4.1 Description of the Computer Setup

The missile system is simulated on a Reeves Electronic Analog Computer (REAC). Outputs are recorded on a Sanborn 4 channel recorder. The idealized system is instrumented, and can be represented by equations (4.1) through (4.5) below. These are rearrangements of equation (1.78) through (1.82), and provide 5 equations in 5 unknowns with each equation written to specify the highest derivative present of one of the variables. The functional layout of the equations is shown in Figure 4.1.*

$$\ddot{\phi} = G(\delta + \delta_T) + F\phi + H\beta + L\gamma \qquad (4.1)$$

$$\dot{\delta} = -(K_4\phi + K_5\dot{\phi} + K_6\ddot{\phi}) \qquad (4.2)$$

$$\dot{\beta} = (A\beta + B\gamma + M\delta) + \sin\alpha_o\dot{\phi} - \cos\alpha_o\dot{\psi} \qquad (4.3)$$

$$\ddot{\psi} = C\beta + E\gamma + N\delta \qquad (4.4)$$

$$\dot{\gamma} = K_1[(A\beta + B\gamma + M\delta) - (A\beta + B\gamma + M\delta)_c] + K_2\dot{\psi} + K_3\ddot{\psi} \qquad (4.5)$$

*Figure D.1, Appendix D shows the actual scaled layout used.

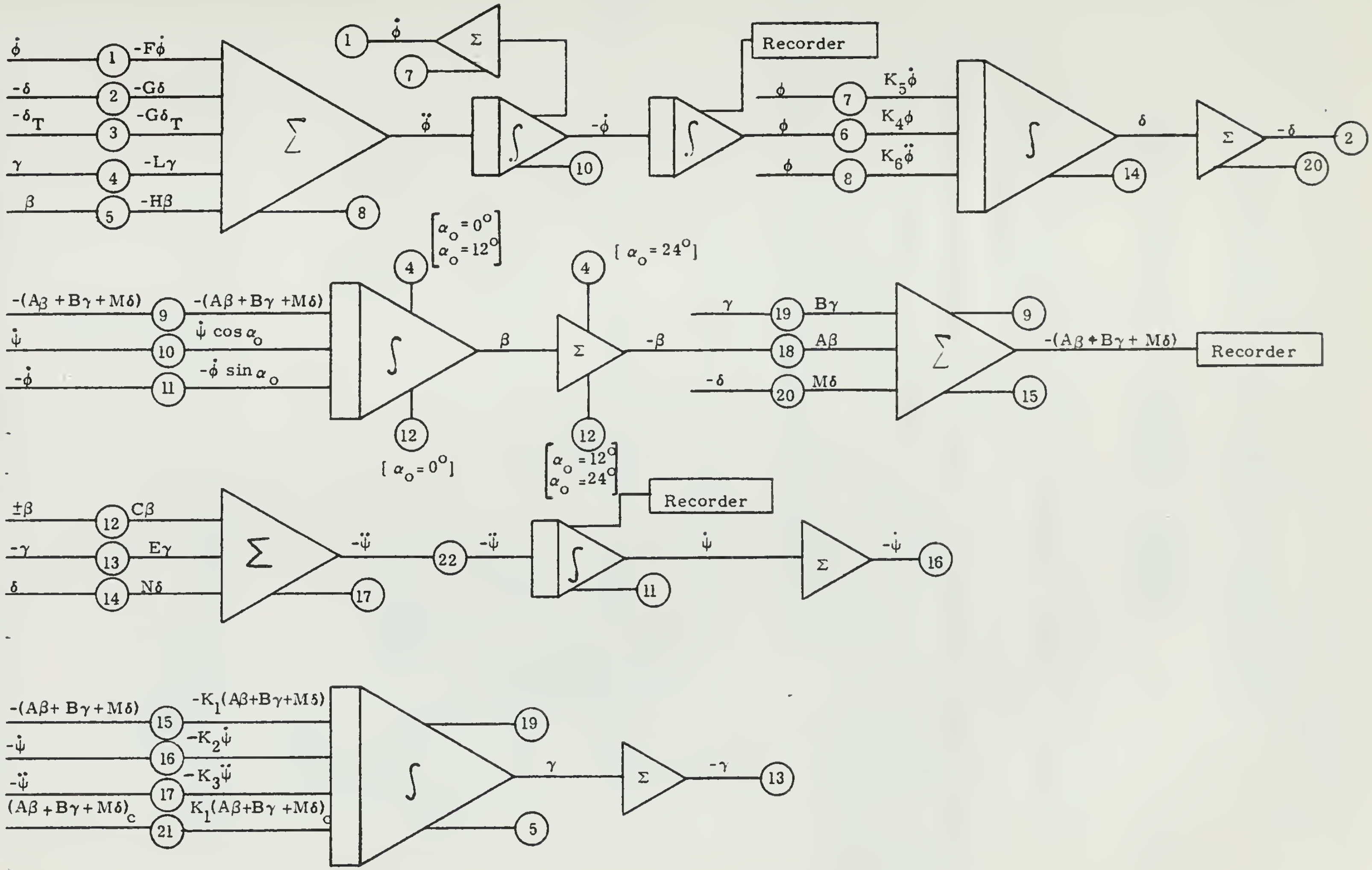

Fig. 4.1 Computer Functional Layout of Idealized System

The system of Figure 4.1 is the one on which all gain variation and adjustment tests are run. In addition, the compensator network of Figure 4.2, representing the approximate compensator,

$$\frac{\delta}{\dot{\psi}}(s) = V(s)\Big]_{24^o} = -\frac{6.71s^2}{s+463} + \frac{1.57s}{s+463} \qquad (4.6)$$

as developed in Section 3.2 is used in testing the effectiveness of decoupling compensation at $\alpha_o > 0$.

Some modifications of the layout of Appendix D are necessary to permit the insertion of the scaled compensator as a transfer function from $\dot{\psi}$ to δ. The modifications are minor, however, and the system is not shown in its modified form. The approximate derivative network is from Johnson, Analog Computer Techniques.[4.1]

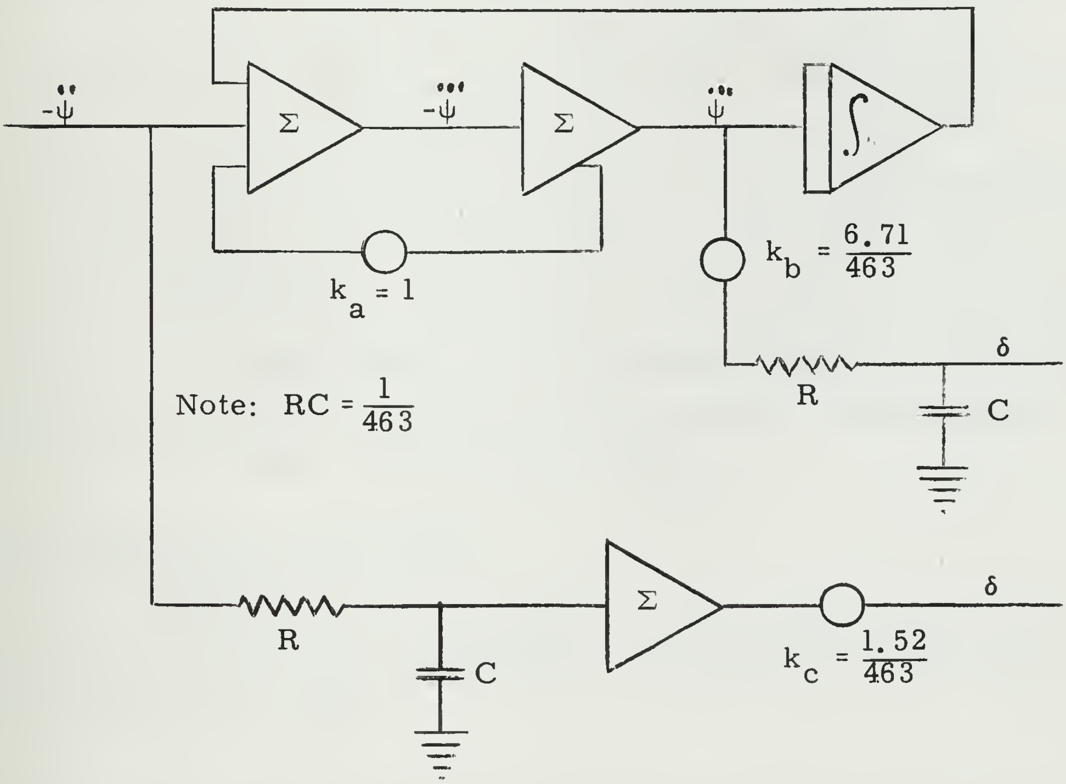

Fig. 4.2 Analogue Computer Simulation of Decoupling Compensator

4.2 Gain Adjustment Techniques

The following gains found by the methods of Section 2.2 to stabilize the system are used as a starting set in adjustment to meet specifications.

$$K_1 = 43.8 \tag{4.7}$$

$$K_2 = 6.18 \tag{4.8}$$

$$K_3 = .357 \tag{4.9}$$

$$K_4 = 27.9 \tag{4.10}$$

$$K_5 = 2.33 \tag{4.11}$$

$$K_6 = .01985 \tag{4.12}$$

Roll adjustment is done by recording and observing ϕ; yaw adjustments made on the recorded step response of $(A\beta + B\gamma + M\delta)$.

By observation of the recorded responses, cyclical gain adjustments in roll and yaw, and at the 3 pitch angles selected are made. It is found that K_3 and K_6 are to be set at or near their maxima. Next adjustments of K_4 and K_5 in roll, K_2 and K_1 in yaw, are made to give specified stable operation of the subsystems with suitable damping of the coupled modes. Subsystem frequencies (and thus rise time) are dependent on K_1 and K_6; damping on K_2 and K_5. These are the rough guides used in obtaining the final gains listed in Section 5.1. Section 5.2 shows system response to step δ_T and to step $(A\beta + B\gamma + M\delta)_c$ at the gains selected.*

4.3 Compensator Simulation

A scaled setup** of the compensator of Figure 4.2 is used in testing the possibility of eliminating or attenuating the effects of a yaw command on roll output.

*Figure D.3, Appendix D shows correlation between analytically derived response of the uncoupled mode and computer response for a common set of gains.

**See Figure D.2, Appendix D.

Responses of the system with and without compensator are compared in Figure 5.7 through 5.9.

CHAPTER 5

RESULTS

5.1 Gains Selected

Final gains selected are:

$$K_1 = 32.0 \tag{5.1}$$

$$K_2 = 6.40 \tag{5.2}$$

$$K_3 = .357 \tag{5.3}$$

$$K_4 = 12.85 \tag{5.4}$$

$$K_5 = 2.33 \tag{5.5}$$

$$K_6 = .0199 \tag{5.6}$$

5.2 System Step Responses

Figures 5.1 through 5.9 are responses of ϕ, $(A\beta + B\gamma + M\delta)$, and $\dot{\psi}$ to step inputs:

$$\delta_T = 1 \text{ degree} \tag{5.7}$$

$$(A\beta + B\gamma + M\delta)_c = 1 \text{ deg/sec} \tag{5.8}$$

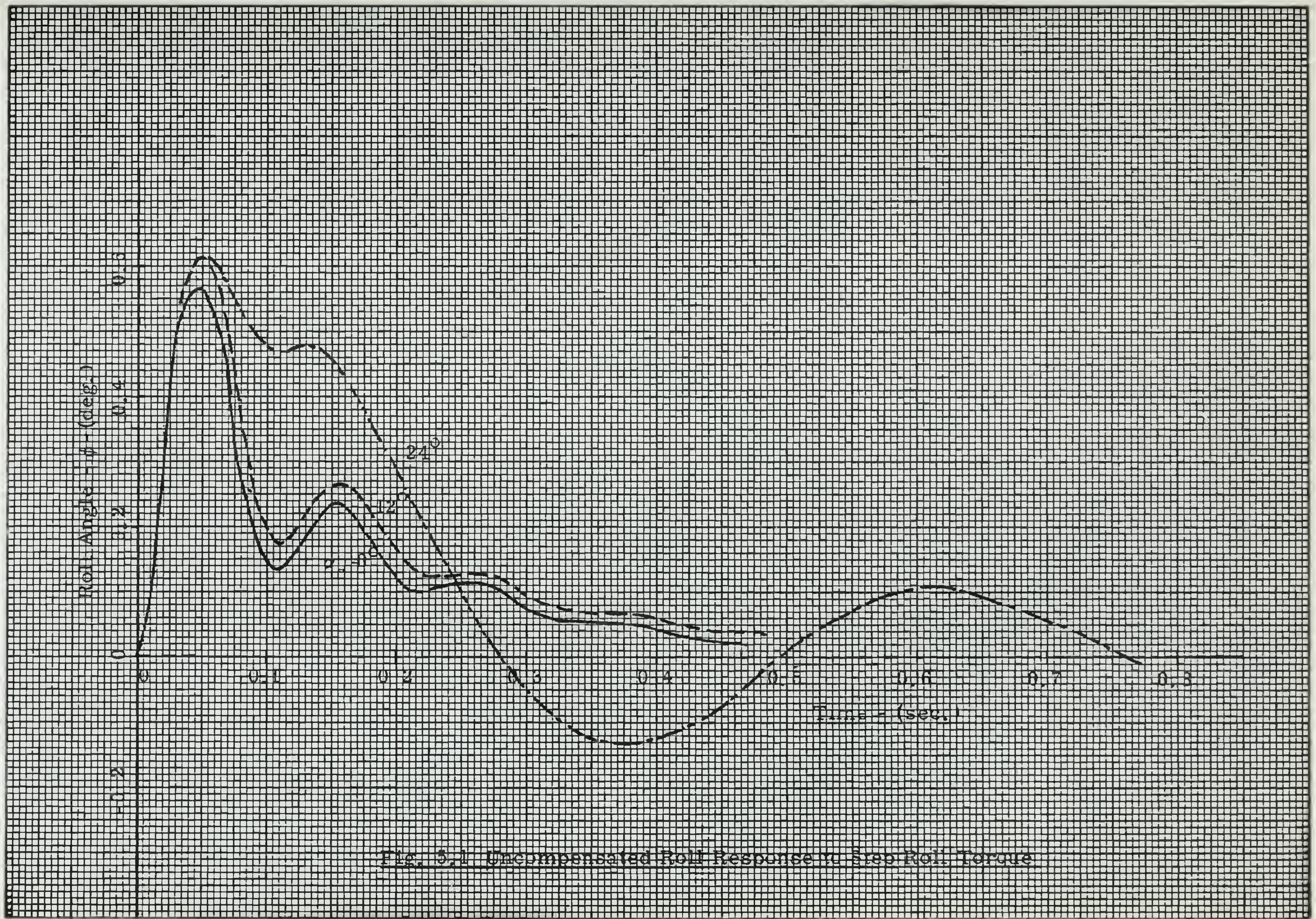

Fig. 5.1 Uncompensated Roll Response to Step Roll Torque

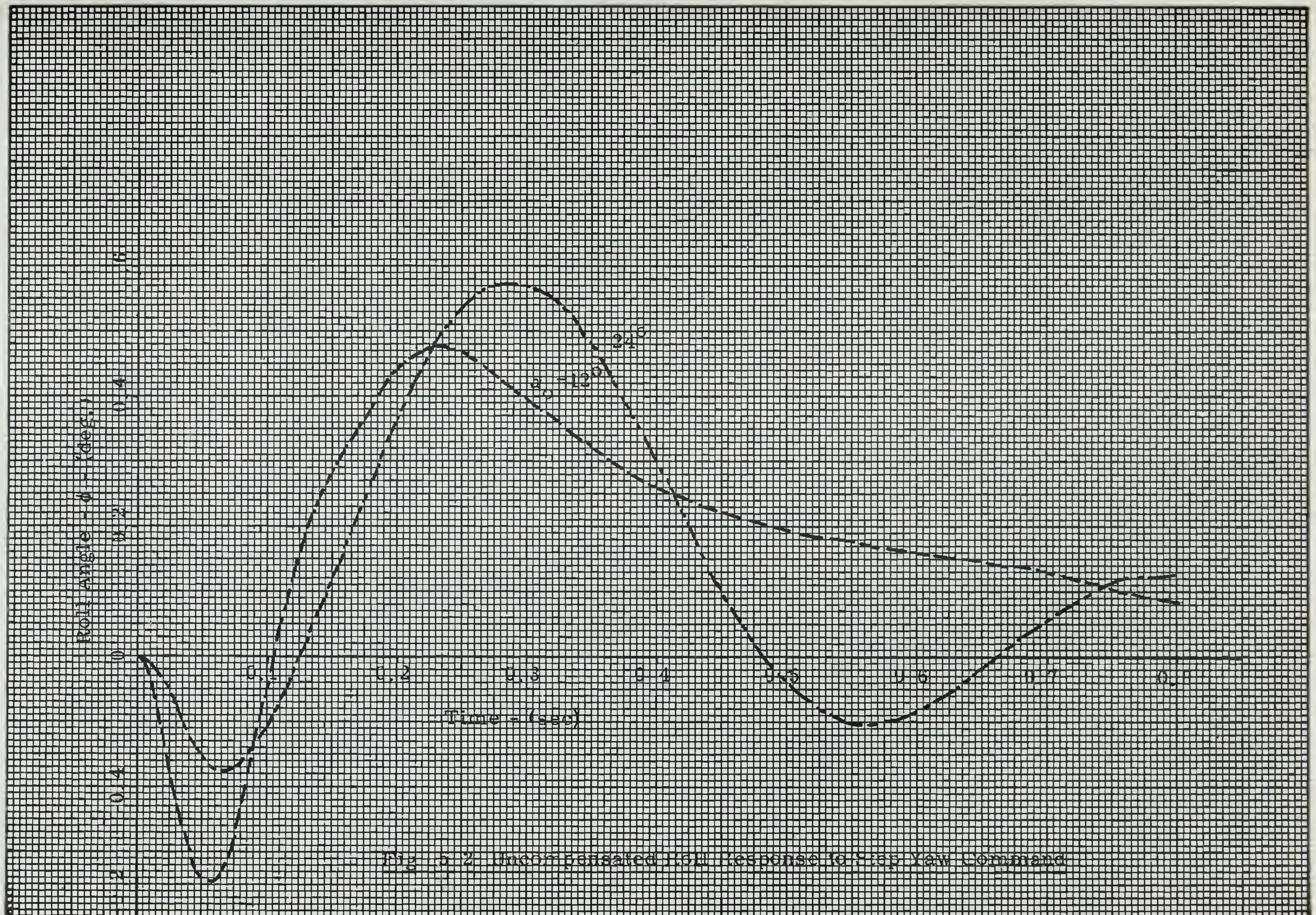

Fig. 5.2 Uncompensated Roll Response to Step Yaw Command

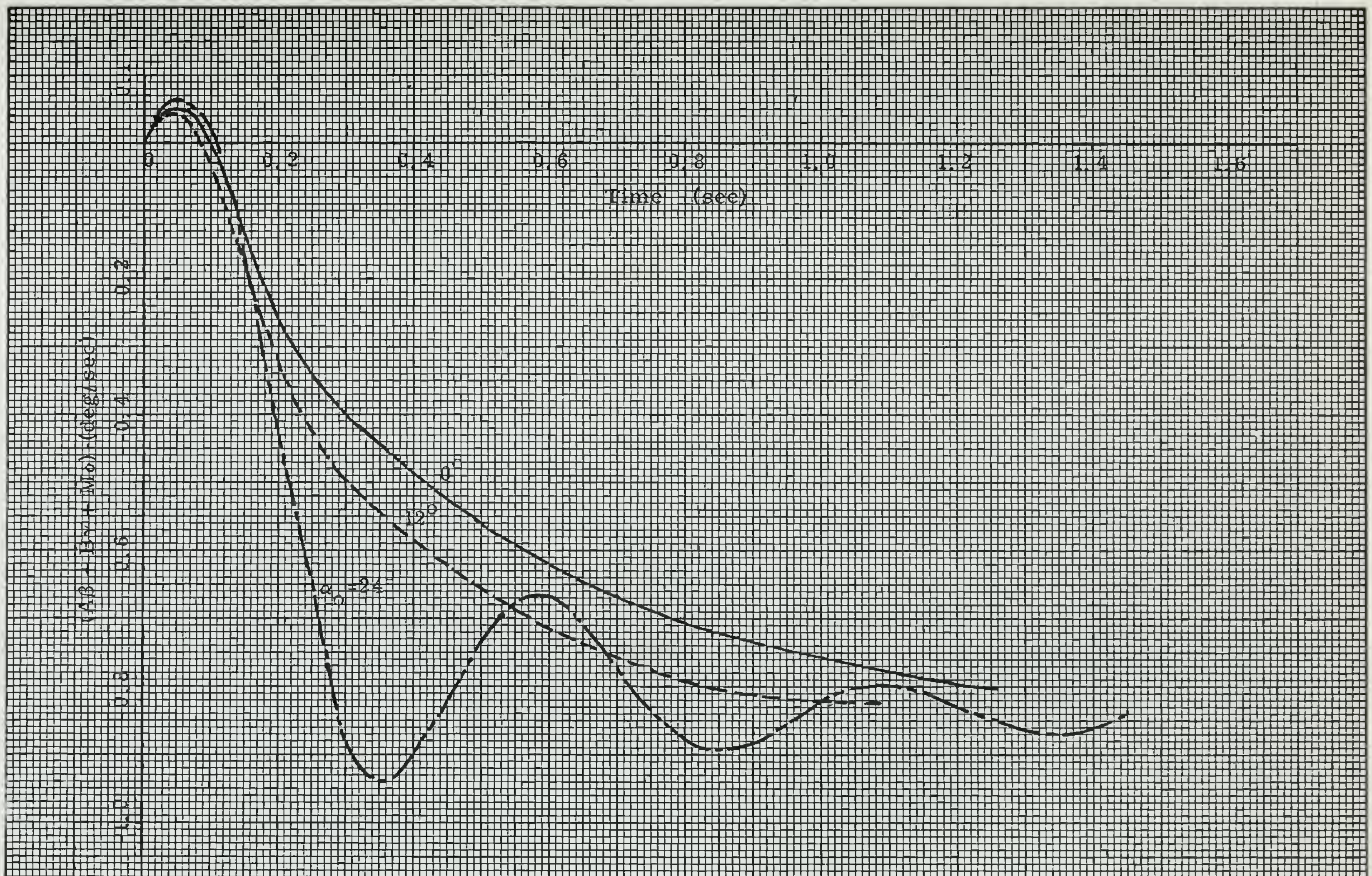

Fig. 5.3 Uncompensated ($A\beta - B\gamma + M\delta$) Response to Step Yaw Command

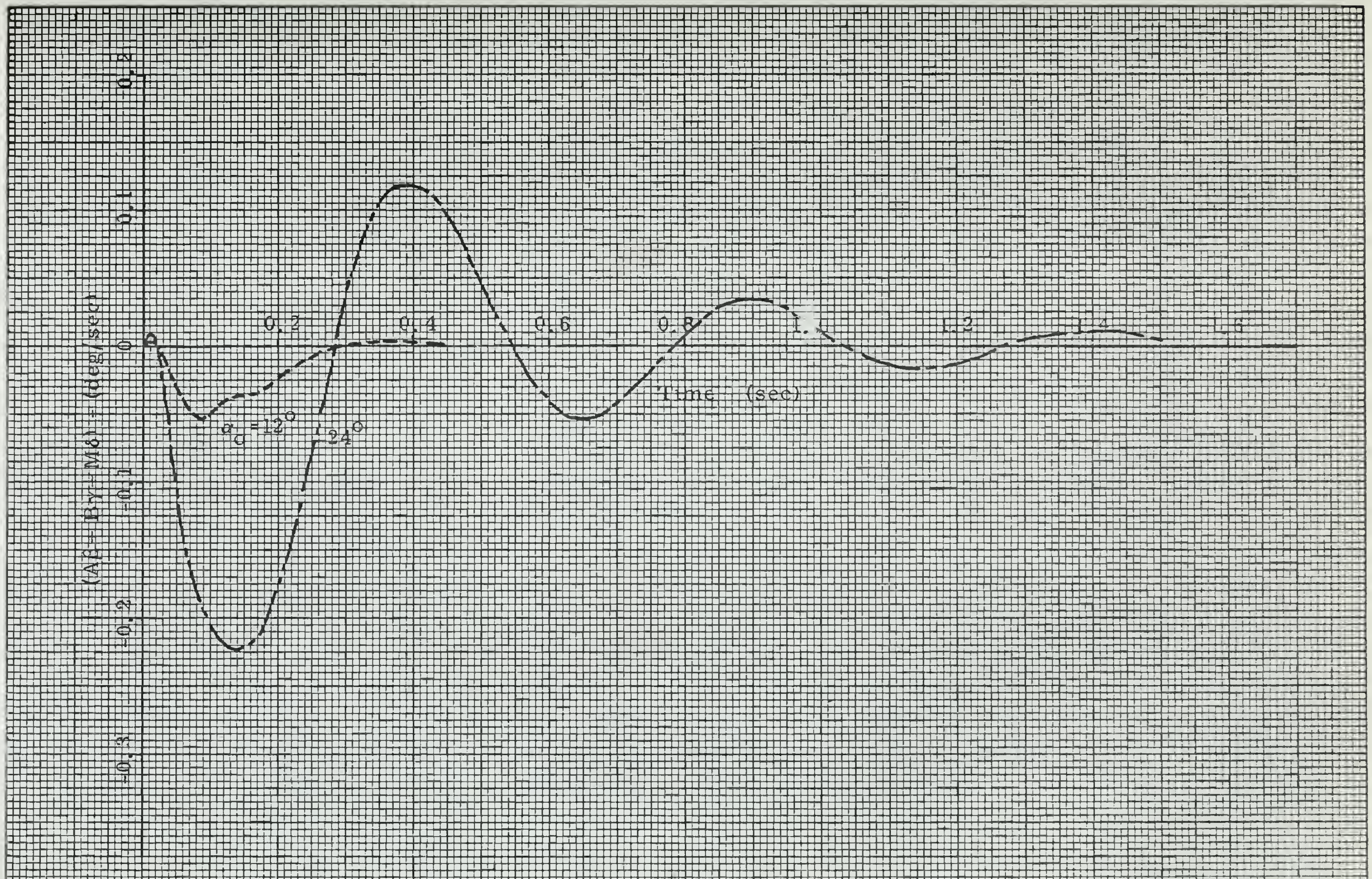

Fig. 5.4 Uncompensated $(A\beta + B\gamma + M\delta)$ Response to Step Roll Command

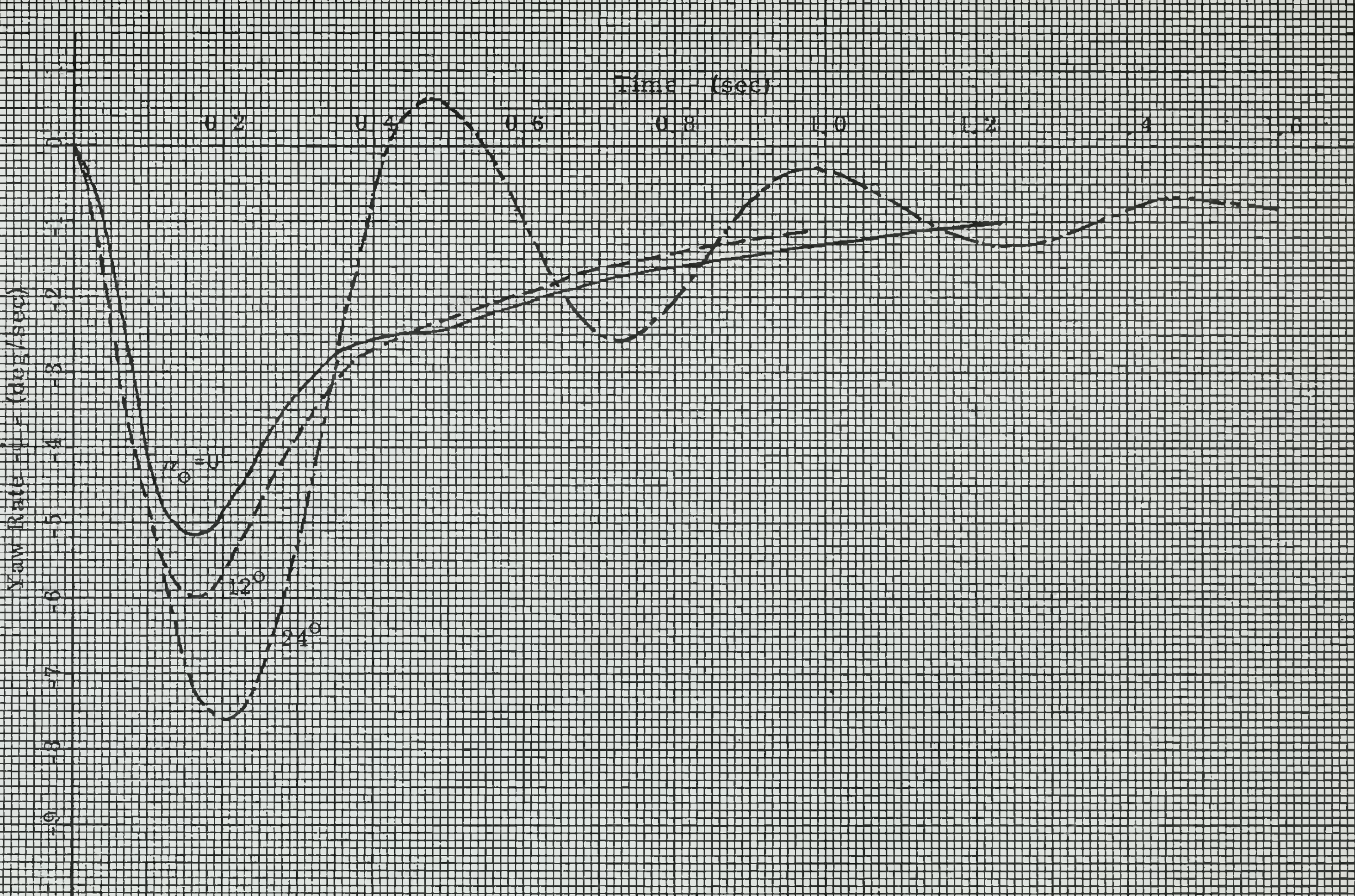

Fig. 5.5 Uncompensated Yaw Rate Response to Step Yaw Command

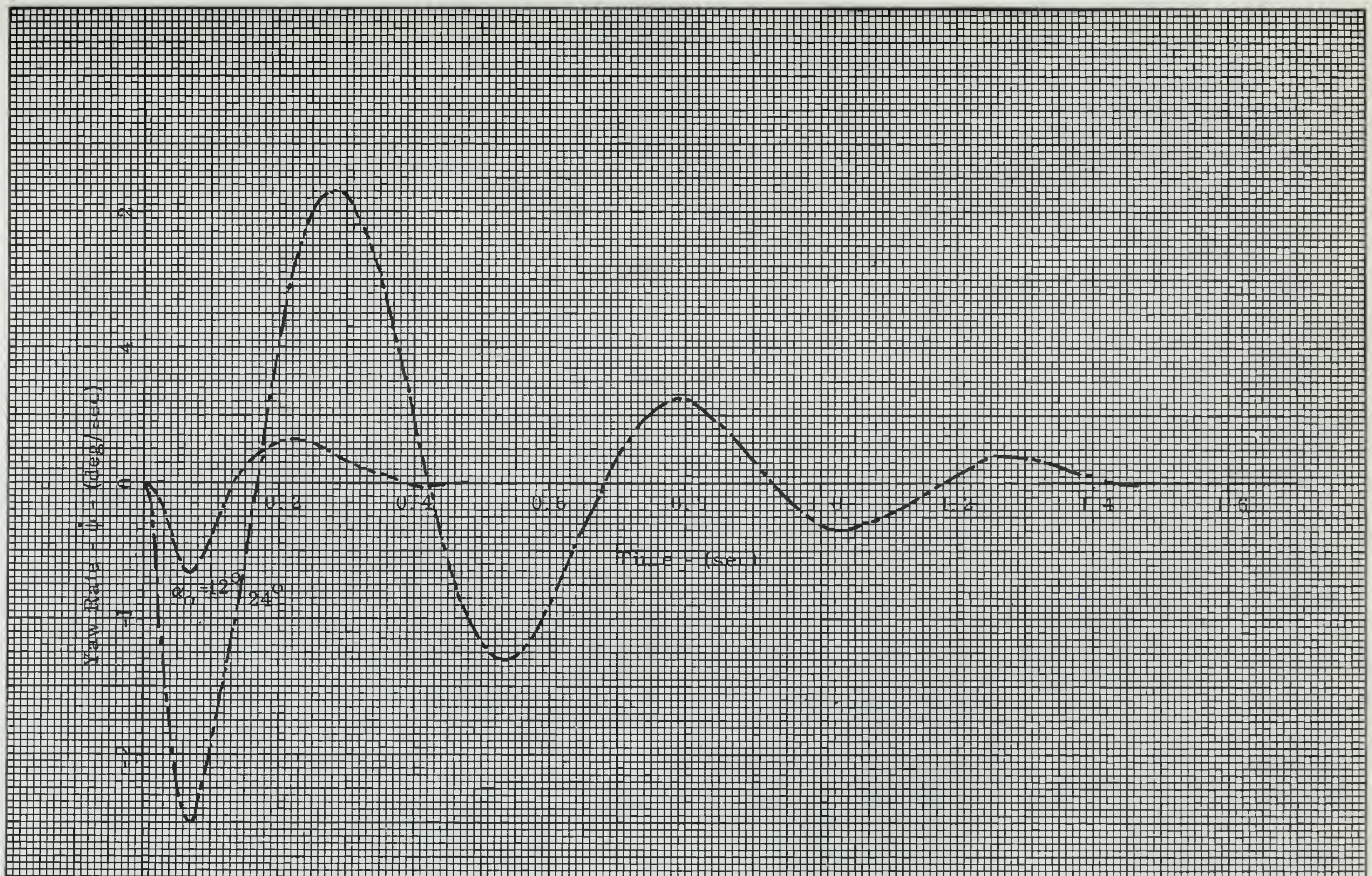

Fig. 5.8 Uncompensated Yaw Rate Response to Step Roll Torque

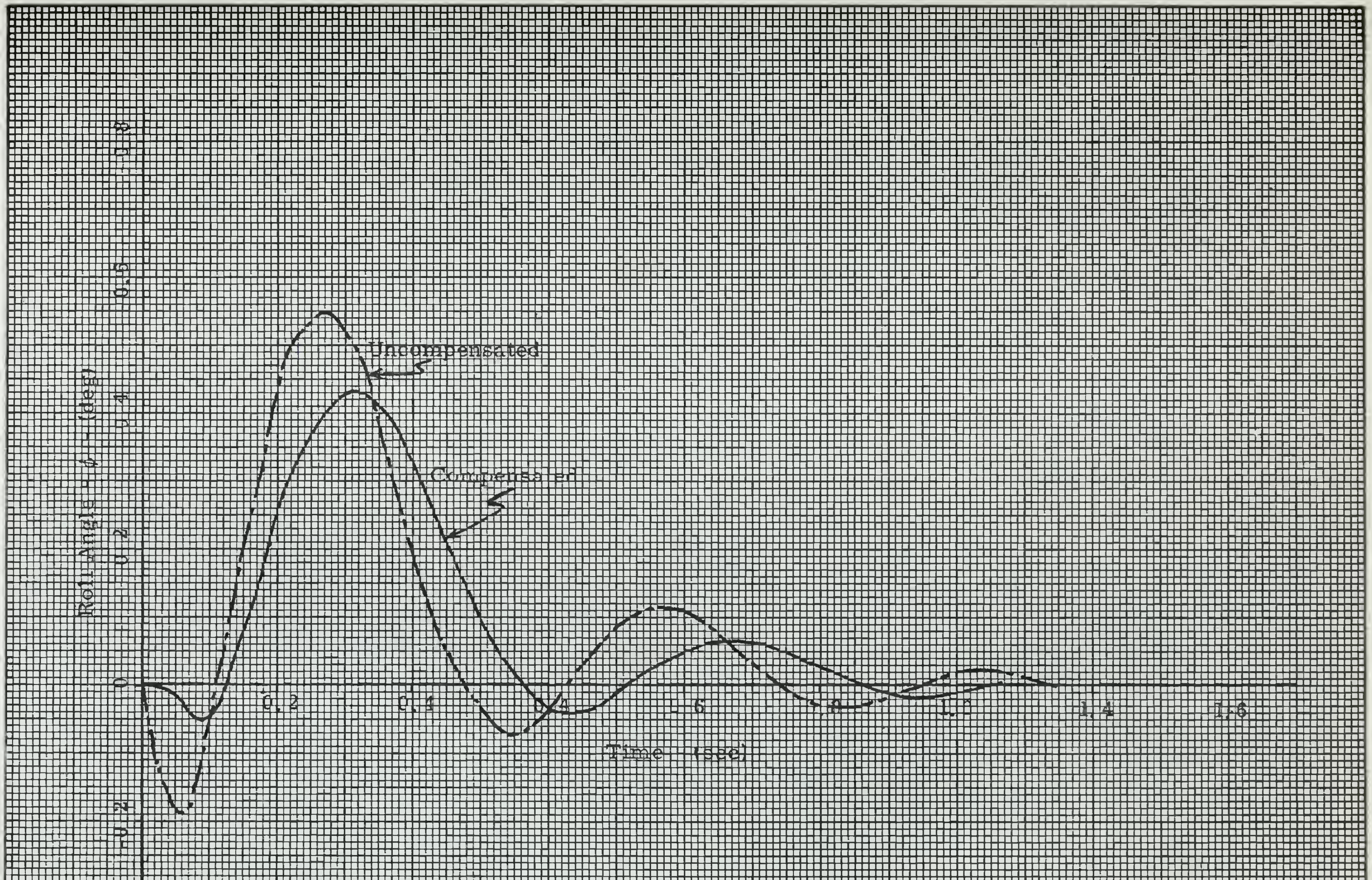

Fig. 5.7 Effect of Compensator on Roll Response to Step
Yaw Command

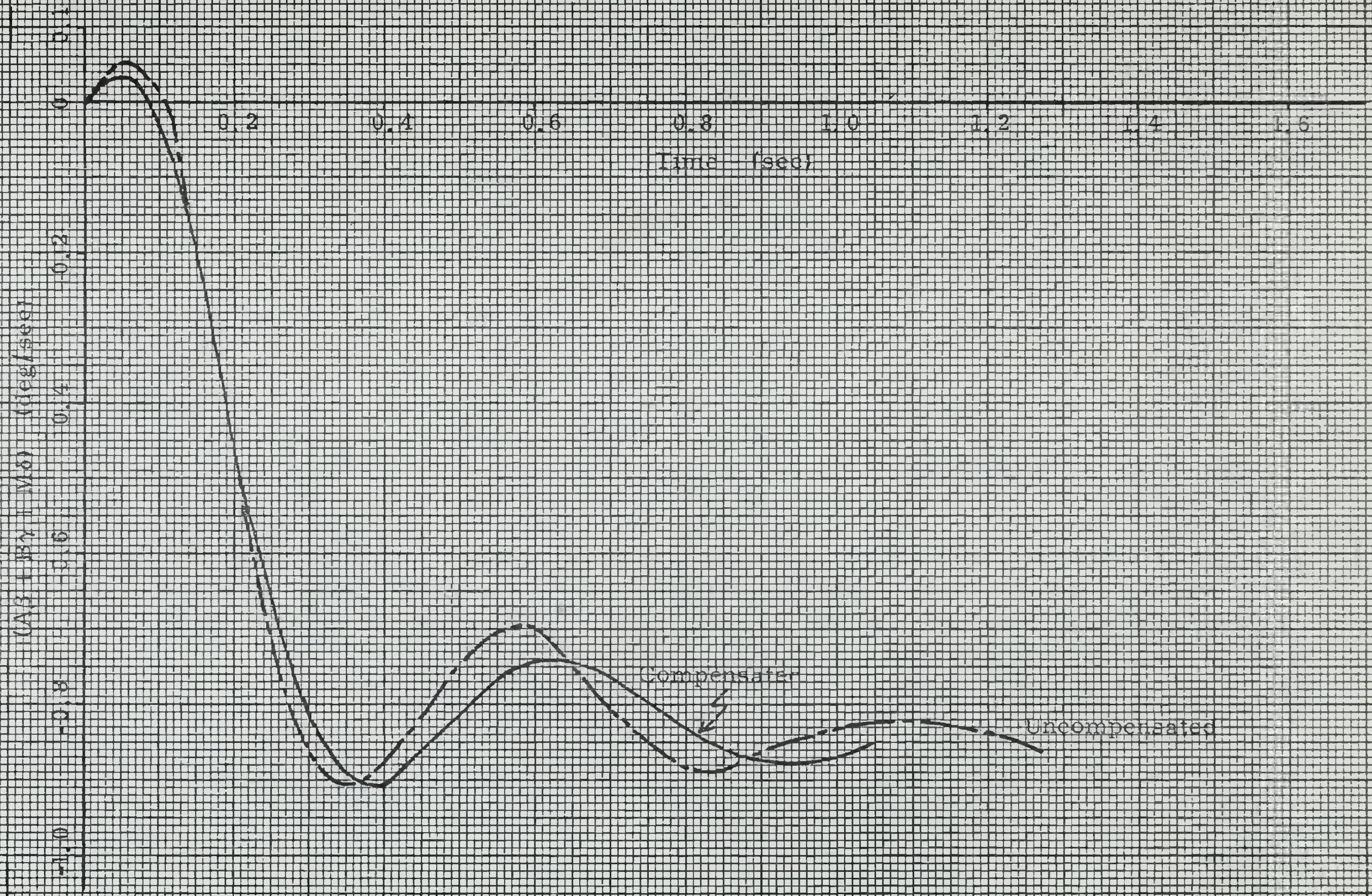

Fig. 5.8 Effect of Compensator on (Aβ + Bγ + Mδ) Response to Step Yaw Command

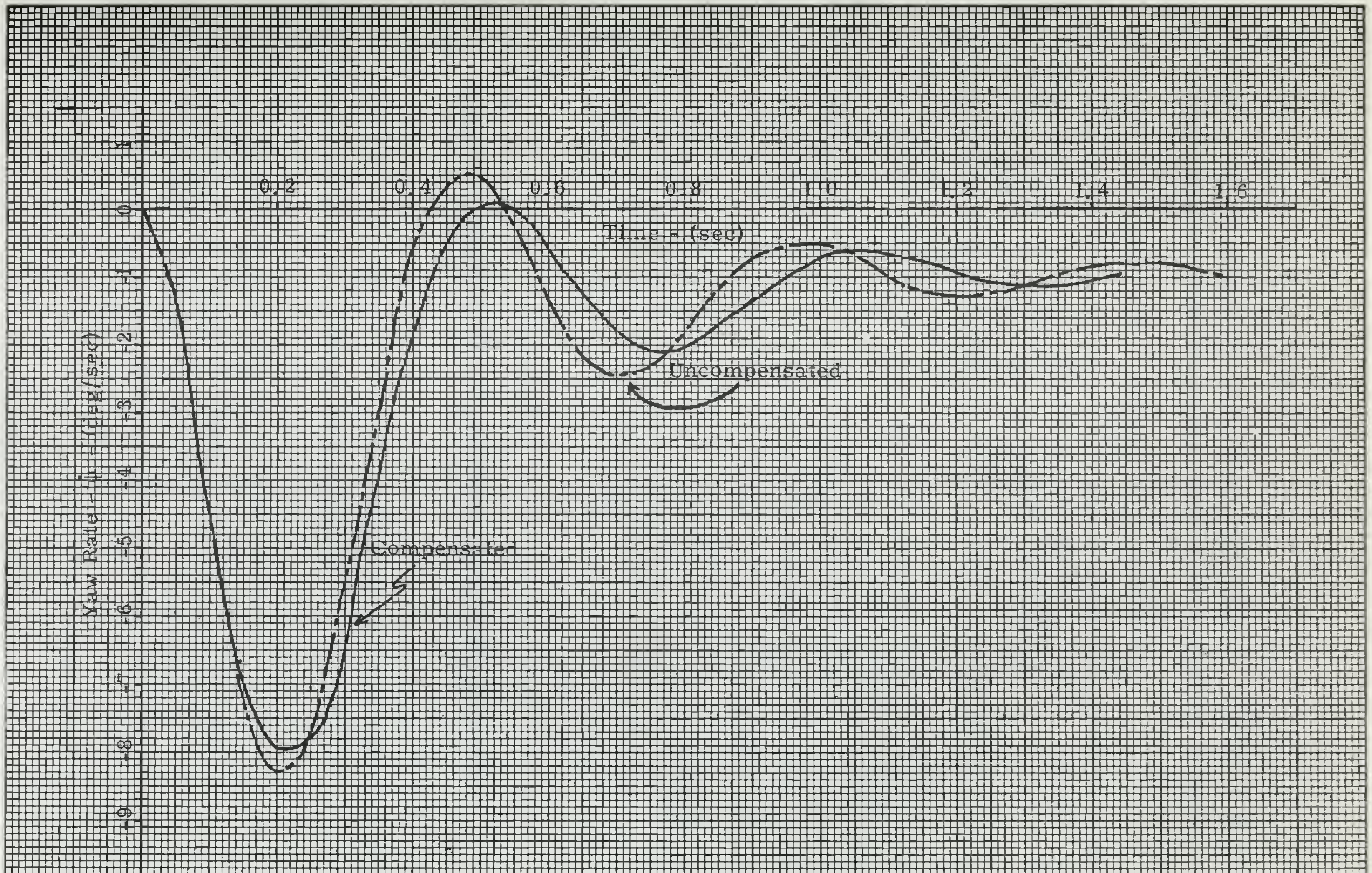

Fig. 5.9 Effect of Compensator on Yaw Rate Response to Step Yaw Command

5.3 System Response Parameters

Reduction of response data from Figures 5.1 through 5.6 gives the uncompensated system response parameters show in Table 5.1.

	Roll		Yaw		
α_o deg	τ^*_{roll} sec.	ϕ_{max} deg.	τ^*_{yaw} sec	$(A\beta + B\gamma + M\delta)_{max}$ $\frac{deg}{sec}$	Input
0	.13	.56		0	$\delta_T = 1^o$
12	.14	.61		.06	
24	.34	.61		.25	
0		0	.44	.84	$(A\beta + B\gamma)_c = 1 \frac{deg}{sec}$
12		.48	.35	.87	
24		.57	.23	.94	

Table 5.1

Response Parameters of the Uncompensated System

Table 5.2 is reduced data from Figures 5.7 through 5.9 and gives parameters of the compensated system at $\alpha_o = 24^o$.

τ^*_{roll} sec	ϕ_{max} deg	τ^*_{yaw} sec	$(A\beta + B\gamma + M\delta)_{max}$ deg/sec
.34	.43	.24	.91

Table 5.2

Response Parameters of the System as Compensated at $\alpha_o = 24^o$

*Roll and yaw subsystem time constants as defined in Section 1.3.

CHAPTER 6

CONCLUSIONS

6.1 Achievement of Specifications

The uncoupled response specifications of Section 1.3 can be readily achieved. At $\alpha_o > 0$, coupling exists which must be eliminated or damped to maintain stable, specified performance. Stable coupled response is obtained at the expense of more sluggish performance in the uncoupled mode.

Satisfactory response in both coupled and uncoupled modes may be obtained, but requires computer simulation for practical investigation and setting of system parameters.

6.2 Elimination of Coupled Response by Compensation

Decoupling by compensation is theoretically possible. However, the compensating devices cannot be achieved with purely passive networks. Also, some physical changes within the control systems are necessary to provide points at which compensating signals may be introduced and extracted. Third, since compensation is a function of angle of attack, there should be some manner of varying the compensator characteristics with angle of attack.

Since some success has been achieved with approximations of theoretical compensators, a more intensive investigation of decoupling compensators might evolve a compensator which is both effective and physically practical.

6.3 Effect of Non-Ideal Servos and Sensors

Since the bandwidths of sensing devices and control surface

servos commonly employed in missiles of this type are of the order of 40 to 50 cps it is clear that the factors of (B. 25) are quite considerably removed from the poles (s-A) and (s-F) and from the origin.

The results of factoring the ideal system characteristic equation indicate that its roots are not far removed from the poles thereof for the gains selected by computer study in Chapter 4. Then by inference, such additional roots as are introduced in the non-ideal system by the presence of control surface servos and sensing devices would be located relatively close to the poles introduced by those devices, and the residues associated with these roots would be small.

Therefore, mathematical representation by the idealized system is considered correct and no gross performance characteristics are thereby over looked. Additionally, as noted elsewhere, available analogue computer facilities do not permit simulation of the servos and sensing devices. Conversation with Professor W. L. Markey of M. I. T. relative to the relation of additional modes introduced by non-ideal instruments to the basic modes of such systems, (especially when these instruments have wide bandwidths compared to the natural modes of the system) would seem to comfirm that this simplifying action is in keeping with common engineering practice.

APPENDIX A

AERODYNAMIC COEFFICIENTS

Values of the aerodynamic coefficients defined by equations (1.53) through (1.62) and evaluated at the selected values of α_o considered in this thesis are shown below in Table A.1. The values were obtained by Convair (Pomona) engineering personnel. Dimensions are such as to make all angles considered to be measured in degrees.

	$\alpha_o = 0^o$	$\alpha_o = 12^o$	$\alpha_o = 24^o$
A	- .373	- .282	- .364
B	+ .0705	+ .0787	+ .0787
C	+8.42	-80.9	-119.5
E	-60.6	-69.0	- 69.0
F	-1.237	- 1.237	- 1.237
G	+1480	+1480	+1590
H	0	-133.5	+1269
L	0	-133.5	-612
M	0	-.0149	-.0526
N	0	+13.3	+46.8

TABLE A.1

AERODYNAMIC COEFFICIENTS

APPENDIX B

SYSTEM CHARACTERISTIC EQUATION

B.1 Ideal System Characteristic Equation

Expansion of the determinant (3.7) gives the ideal system characteristic equation in symbolic form

$$\Delta = 1 - F_{13} F_5 F_3 - F_{12} F_1 F_6 - F_3 F_{12} - F_1 F_5 - F_{13} F_6$$

$$+ F_2 [-F_8 - F_{13} F_5 F_9 - F_9 F_{12} + F_8 F_{13} F_6]$$

$$+ F_{11} F_{13} [F_5 F_{10} - F_5 F_4 F_9 - F_7 F_8 F_3 - F_9 F_7 + F_8 F_4 F_6 + F_{10} F_5 F_3]$$

$$+ F_{11} [-F_{10} - F_{12} F_4 F_9 - F_{14} F_8 F_3 - F_9 F_{14} - F_8 F_4 + F_{10} F_{12} F_3]$$

$$+ F_{11} F_1 [-F_5 F_{10} - F_{12} F_7 F_9 - F_{14} F_8 F_6 + F_9 F_5 F_{14} - F_8 F_7 + F_{10} F_{12} F_6]$$

Upon substitution for the defined transfer functions and obtaining a common denominator for all terms except the first (unity) there results:

$$\begin{aligned}
\Delta = 1 &+ \frac{-CB\,(K_3 s+K_2)(s-F)s^2 + K_1 AE\cos\alpha_o\,(s-F)s^2 - BK_1A\,(s-F)s^3}{s^3\,(s-A)(s-F)(s-K_1B)} \\
&+ \frac{C\,\cos\alpha_o(s-F)(s-K_1B)s^2 - E(K_3s+K_2)(s-A)(s-F)\,s^2 - H\,\sin\alpha_o\,(s-K_1B)\,s^3}{s^3\,(s-A)(s-F)(s-K_1B)} \\
&+ \frac{-LC\sin\alpha_o(K_3s+K_2)(s-A)s^2 - LK_1A\sin\alpha_o(s-A)s^3 + HE\,\sin\alpha_o(K_3s+K_2)(s-A)s^2}{s^3(s-A)(s-F)(s-K_1B)} \\
&+ \frac{-(K_6s^2+K_5s+K_4)(K_3s+K_2)[GE(s-A)(s-K_1B)s^2 - (LMC+NHB)(s-K_1B)s^2]}{s^3\,(s-A)(s-F)(s-K_1B)} \\
&+ \frac{-(K_6s^2+K_5s+K_4)(K_3s+K_2)[-LN(s-K_1B)(s-A)s^2+(HME+GCB)(s-K_1B)s^2]}{s^3(s-A)(s-F)(s-K_1B)} \\
&+ \frac{-(K_6s^2+K_5s+K_4)[G(s-A)(s-K_1B)s - (K_1MHB)s - LK_1M(s-A)s - HM(s-K_1B)s + (K_1GAB)s]}{s^3\,(s-A)(s-F)(s-K_1B)} \\
&+ \frac{\cos\alpha_o(K_6s^2+K_5s+K_4)[-GC(s-K_1B)-K_1ANL-K_1MHE + LCMK_1 - HN(s-K_1B) + GK_1AE]}{s^3\,(s-A)(s-F)(s-K_1B)}
\end{aligned} \tag{B. 2}$$

Upon substituting the aerodynamic coefficients evaluated at $\alpha_o = 24^o$, the C.E. becomes:

$$\Delta = - s^2[516\ s^2 + (50\ K_1 + 65,400\ K_3)s + 65,400\ K_2]$$

$$- s^2(s+1.237)[-s^3 + (-69\ K_3 + .0787\ K_1 - .364)\ s^2 - (103.5\ K_2)$$

$$+ (K_6\ s^2 + K_5\ s + K_4)[-120,000\ s + 37,200\ K_1]$$

$$+ s^2\ (K_6\ s^2 + K_5\ s + K_4)[1590\ s + (-92.8\ K_1 + 511.3)]$$

$$+ (K_6\ s^2 + K_5\ s + K_4)(K_3\ s + K_2)(81,200\ s + 40,650) \qquad \text{(B. 3)}$$

B. 2 Non-Ideal system characteristic equation

To obtain the characteristic equation of the coupled system when non-ideal sensing devices and non-ideal control surface servos are employed it is useful to note that these affect only the "roll control" and "yaw control" equations of (3.1).

The "roll control" equation is modified to

$$\delta(s) = - G_5(K_6 s^2 + K_5 s + K_4)G_2 \quad (\frac{1}{s^2})\ \dot{\phi}\,(s) \qquad \text{(B. 4)}$$

The "yaw control" equation is modified to

$$s\gamma(s) = G_4[K_1(A\beta + B\gamma + M\delta)G_1 + G_3(K_3 s + K_2)\dot{\psi}\,] \qquad \text{(B. 5)}$$

When these are case in the form employed in (3.1) the system determinant becomes

$$
\begin{array}{c}
\begin{array}{ccccc}
\beta & \dot{\psi} & \dot{\phi} & \gamma & \delta
\end{array}\\
\left|\begin{array}{ccccc}
s-A & \cos\alpha & -\sin\alpha & -B & -M \\
-C & s & 0 & -E & -N \\
-H & 0 & s-F & -L & -G \\
0 & 0 & \dfrac{G_2G_5X}{s^2} & 0 & 1 \\
(-K_1G_4G_1A) & (-G_3G_4Y) & 0 & (s-K_1G_4G_1B) & (-KG_4G_1M)
\end{array}\right|
\end{array}
\tag{B. 6}
$$

where G_1 through G_5 are as defined in Chapter 1 Section 1. 2. This constitutes a modification of the transfer functions defined in Chapter 3 based on these two equations. The following modified transfer functions are therefore defined to replace the corresponding transfer functions of Chapter 3.

$$\frac{\delta}{\dot{\phi}}(s) = -\frac{G_2G_5X}{s^2} \equiv F_{11a} \tag{B. 7}$$

$$\frac{\gamma}{\beta}(s) = \frac{K_1G_4G_1A}{(s-K_1G_4G_1B)} \equiv F_{12a} \tag{B. 8}$$

$$\frac{\gamma}{\dot{\psi}}(s) = \frac{G_3G_4Y}{(s-K_1G_4G_1B)} \equiv F_{13a} \tag{B. 9}$$

$$\frac{\gamma}{\delta}(s) = \frac{K_1G_4G_1M}{(s-K_1G_4G_1B)} \equiv F_{14a} \tag{B. 10}$$

$$\frac{\gamma}{(A\beta + B\gamma)_{\varsigma}}(s) = \frac{-K_1G_4G_1}{(s-K_1G_4G_1B)} \equiv F_{aa} \tag{B. 11}$$

With these modifications the same block diagrams, Figures 3.6 and 3.7, can be used and the expanded form of (3.7), see (B.1), be readily modified. For this purpose it is convenient to regroup the terms of (B.1) as follows:

$$\Delta = 1 - F_{13a}[-F_5F_3 - F_6 - F_2F_5F_9 + F_2F_8F_6]$$

$$+ F_{12a}[-F_1F_6 - F_3 - F_2F_9]$$

$$+ F_{11a}F_{13a}[F_6F_{10} - F_5F_4F_9 - F_7F_8F_3 - F_9F_7 + F_8F_4F_6 + F_{10}F_5F_3]$$

$$+ F_{11a}[-F_{10} - F_8F_4 - F_1F_5F_{10}]$$

$$+ F_{11a}F_{12a}[-F_4F_9 + F_{10}F_3 - F_1F_7F_9 + F_{10}F_1F_6]$$

$$+ F_{11a}F_{14a}[-F_8F_3 - F_9 - F_1F_8F_6 + F_1F_9F_5]$$

$$- F_1F_5 - F_2F_8 - F_7F_8 \qquad \text{(B.12)}$$

Upon substitution for all defined transfer functions except F_{11} through F_{14} there results

$$\Delta = 1 - F_{13a}\left[-\frac{C\ \ B}{s\,(s-A)} - \frac{E}{s} - \frac{C \sin\alpha_o\ \ L}{s\,(s-A)\,(s-F)} + \frac{E \sin\alpha_o\ \ H}{s\,(s-A)\,(s-F)} \right]$$

$$+ F_{12a}\left[\frac{E \cos\alpha_o}{s\,(s-A)} - \frac{B}{(s-A)} - \frac{\sin\alpha_o\ \ L}{(s-A)\,(s-F)} \right]$$

$$+ F_{11a}F_{13a}\left[\frac{E\ \ G}{s\,(s-F)} - \frac{C\ \ M\ \ L}{s\,(s-A)(s-F)} - \frac{N\ \ B\ \ H}{s\,(s-A)(s-F)} - \frac{N\ \ L}{s\,(s-F)} + \frac{E\ \ M\ \ H}{s\,(s-A)(s-F)} + \frac{C\ \ B\ \ G}{s\,(s-A)(s-F)} \right]$$

$$+ F_{11a}\left[-\frac{G}{(s-F)} - \frac{M\ \ H}{(s-A)(s-F)} + \frac{C\,G \cos\alpha_o}{s\,(s-A)(s-F)} \right]$$

$$+ F_{11a}F_{12a}\left[-\frac{M\ \ L}{(s-A)(s-F)} + \frac{B\ \ G}{(s-A)(s-F)} + \frac{N \cos\alpha_o\ \ G}{s\,(s-A)(s-F)} - \frac{E\ \ G\ \cos\alpha_o}{s\,(s-A)(s-F)} \right]$$

$$+ F_{11a}F_{14a}\left[-\frac{B\ \ H}{(s-A)(s-F)} - \frac{L}{(s-F)} + \frac{E \cos\alpha_o\ \ H}{s\,(s-A)(s-F)} - \frac{C \cos\alpha_o\ \ L}{s\,(s-A)(s-F)} \right]$$

$$+ \frac{C \cos\alpha_o}{s\,(s-A)} - \frac{\sin\alpha_o\ \ H}{(s-A)(s-F)} - \frac{N\ \ H}{s\,(s-F)}$$

Since the control surface servos and the sensors are presumed* to be second order devices, it is possible to assess the effects of the modified transfer functions F_{11a} through F_{14a} as follows:

Define

$$G_1 \equiv \frac{b}{s^2 + as + b} \tag{B.14}$$

$$G_2 \equiv \frac{d}{s^2 + cs + d} \tag{B.15}$$

$$G_3 \equiv \frac{f}{s^2 + es + f} \tag{B.16}$$

$$G_4 \equiv \frac{h}{s^2 + gs + h} \tag{B.17}$$

$$G_5 \equiv \frac{n}{s^2 + ms + n} \tag{B.18}$$

Then the modified transfer functions become

$$F_{11a} = \frac{d}{(s^2 + cs + d)} \cdot \frac{n \quad X}{(s^2 + ms + n)\ s^2} \tag{B.19}$$

$$F_{12a} = \frac{K_1 A \dfrac{h}{(s^2 + gs + h)} \cdot \dfrac{b}{(s^2 + as + b)}}{s - K_1 B \dfrac{h}{(s^2 \Delta\ gs + h)} \cdot \dfrac{b}{(s^2 + as + b)}} \tag{B.20}$$

$$= \frac{K_1 A\, hb}{[s\,(s^2 + gs + h)(s^2 + as + b) - K_1 B\, hb]} \tag{B.21}$$

*Telephone conversation with Mr. R. L. Geisberg, CVAC, Pomona.

$$F_{13a} = \frac{\dfrac{f \qquad h}{(s^2 + es + f)\,(s^2 + gs + h)}}{\dfrac{s(s^2 + gs + h)\,(s^2 + as + b) - K_1 B\,hb}{(s^2 + gs + h)(s^2 + as + b)}}$$

$$= \frac{f\,h\,(s^2 + as + b)}{[\,s\,(s^2 + gs + h)(s^2 + as + b) - K_1 Bhb]\,(s^2 + es + f)} \tag{B. 22}$$

$$F_{14a} = \frac{K_1 M \dfrac{h}{(s^2 + gs + h)} \cdot \dfrac{b}{(s^2 + as + b)}}{\dfrac{[\,s\,(s^2 + gs + h)(s^2 + as + b) - K_1 Bhb]}{(s^2 + gs + h)(s^2 + as + b)}}$$

$$= \frac{K_1 Mhb}{[s\,(s^2 + gs + h)(s^2 + as + b) - K_1 Bhb]} \tag{B. 24}$$

From these it is clear that the common denominator of equation (B.13) will contain the product.

$$s^3(s^2+cs+d)(s^2+ms+n)[s(s^2+gs+h)(s^2+as+b)-K_1Bhb]\,(s^2+es+f)(s-A)(s-F) \tag{B. 24}$$

Of these poles of Δ, $s^3(s-A)(s-F)$ are as in the ideal system. The other poles are due to the factors of

$$(s^2+cs+d)(s^2+ms+n)[s(s^2+gs+h)(s^2+as+b) - K_1Bhb]\,(s^2+es+f) \tag{B. 25}$$

APPENDIX C

CHU'S METHOD

It is the purpose of this appendix to briefly describe the procedures by which the "system determinant" employed in Chapters 2 and 3 is derived from the "standard block diagram". A complete development of these standardized manipulations as well as a considerable body of analysis and design theory for multiple loop systems can be found in the works of Chu[C.1] and Thaler[C.2].

It is clear from the manipulation performed in the early part of Chapter 3 that the principle of block diagram representation of a set of linear integro-differential equations is simply one of expressing in LaPlace notation the relations in each equation as a summation process wherein several quantities are employed to form a single output quantity from a node. This process is repeated for each equation, and finally, interconnections between identical variables are made to complete the block diagram. This procedure is in common use.

From the intermediate block diagram thus derived, the system characteristic equation can be gotten by any one of a number of methods including that due to Mason[C.3] as noted in Chapters 2 and 3, however, the method developed by Chu[C.1] provides additional facility in performing analysis for each of the several system variables and in design of compensation.

Figure C.1 is the block diagram of an "n node system" (n equations in n variables). The diagram permits of consideration of the introduction of any number of inputs (forcing functions) to any combination of nodes. Any block diagram derived by the common process stated above can be recast into the "standardized" form by simply listing the nodes from left to right in any sequence and performing the simple diagram manipulations necessary to accommodate the following conventions:

a. Each nodal point may receive any number of signals, but delivers only one output.

b. Signals arriving at a node from a node to the left (feed forward) are arbitrarily considered to enter a node additively. (Sign adjustment in the feed forward transfer function may be necessary to accommodate this convention.)

c. Signals arriving at a node from a point to the right (feedback) are arbitrarily considered to enter a node subtractively. (Sign adjustment in the feedback transfer function may be necessary to accommodate this convention.)

d. Signals to be fed forward are derived at a connection immediately to the right of a node.

e. Signals to be fed back are derived at a connection immediately to the left of a node. (This and (d) frequently require moving a "pickoff point" where a signal actually exists in the basic block diagram to the right or left past a transfer function block.)

Referring to Figure C.1 and considering the equilibrium equality at node a we have, for the sign conventions noted above:

$$a = -(G_{ab}G_{Aa})a - (G_{bc}G_{Ba})b - \ldots\ldots -(G_{no}G_{Na})n \qquad \text{(C.1)}$$

$$+ G_{1a}\,I + G_{2a}\,II + \ldots\ldots\; G_{na}\,N$$

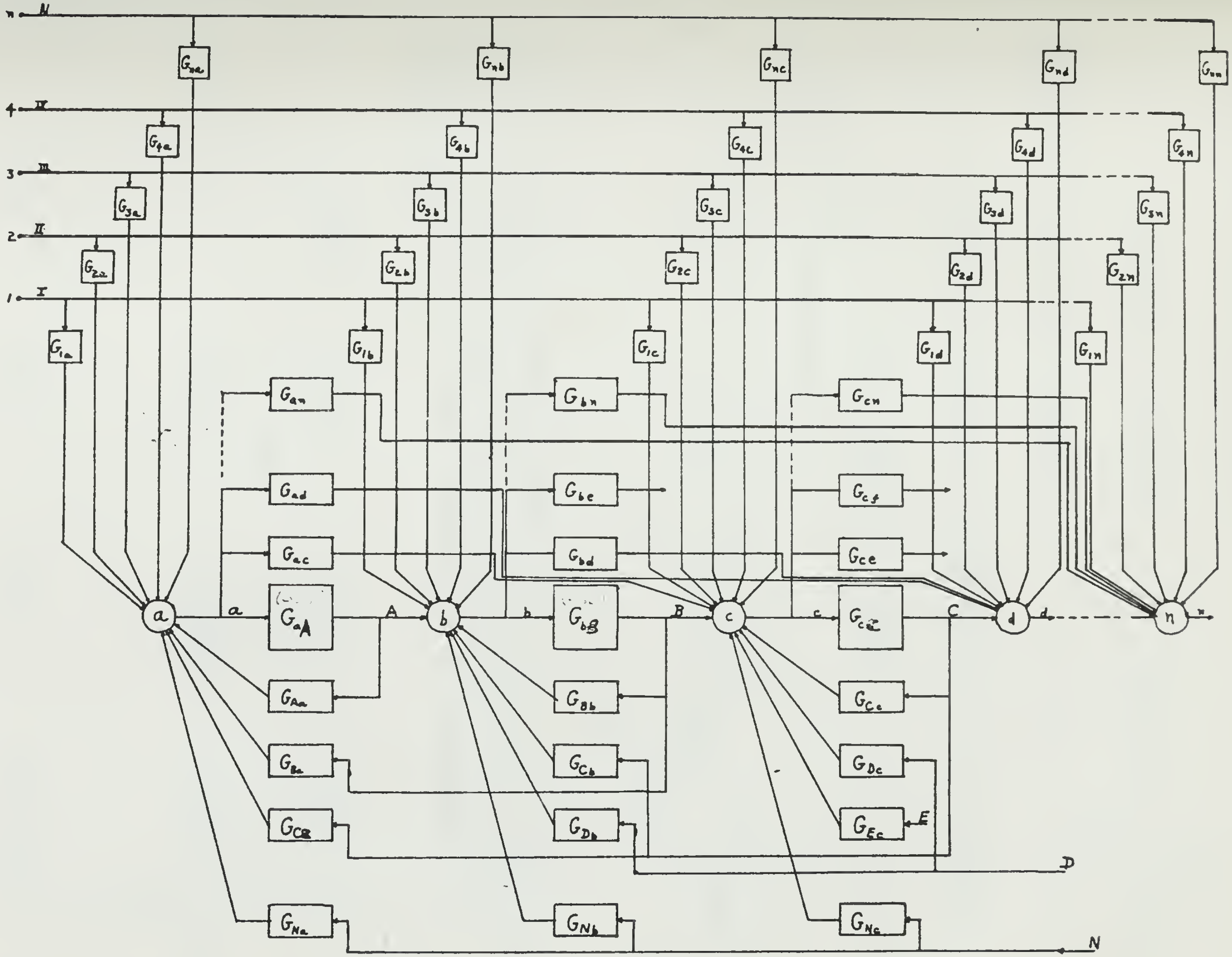

Fig. C.1 Standardized Block Diagram of an n Node System

This equation can be rewritten as:

$$(1 + G_{ab}G_{Aa})a + (G_{bc}G_{Ba}) + \ldots\ldots + (G_{no}G_{Na})n = \sum_{m=I}^{m=N} (G_{ma})m \tag{C.1a}$$

At node b we have the equality:

$$b = (G_{ab})a - (G_{bc}G_{Bb})b - (G_{cd}G_{Cb})c - \ldots\ldots - (G_{no}G_{Nb})n$$

$$+ G_{1b}I + G_{2b}II + \ldots\ldots G_{nb} \tag{C.2}$$

which can be rearranged to give:

$$-(G_{ab})a + (1 + G_{bc}G_{Bb})b + (G_{cd}G_{Cb})c + \ldots\ldots + (G_{no}G_{Nb})n$$

$$= \sum_{m=I}^{m=N} (G_{mb})m \tag{C.2a}$$

Extension of this process to several nodes using care to maintain the sequence established in (C.1a) and (C.2a) it can be seen that the sequence of nodal equations of equilibrium can be expressed in a determinant:

$$\Delta = \begin{vmatrix} 1+G_{aA}G_{Aa} & G_{bc}G_{Ba} & G_{cd}G_{Ca} \cdots & G_{no}G_{Na} \\ -G_{ab} & 1+G_{bc}G_{Bb} & G_{cd}G_{Cb} \cdots & G_{no}G_{Nb} \\ -G_{ac} & -G_{bc} & 1+G_{cd}G_{Cc} \cdots & G_{no}G_{Nc} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ -G_{an} & -G_{bn} & -G_{cn} & 1+G_{no}G_{Nn} \end{vmatrix} \tag{C.3}$$

Inspection of (C.3) leads to the conclusion that, given the block diagram in standard form it is possible to write down the elements of the system determinant by inspection of the diagram, using only so much of the n node determinant pattern as applies to the system at hand. The main

diagonal divides feedback and feed forward regions and terms of the main diagonal are the characteristic equations of the interior loops at the node indicated by the row and column of the main diagonal element. Expansion of the determinant in any given case produces the system characteristic equation without the often confusing mental bookkeeping necessary with some other methods, including that of Mason[C. 3].

By suitable choice of the method of selecting the sequence of nodes for drawing the "standard block diagram" it is possible (as in Chapter 3) to graphically illustrate in the determinant the couplings that exist within the original equations. Solution of the equations for desired input - output relations follows Cramer's Rule, the column into which the forcing functions are substituted, representing the node in the standard block diagram at which the desired variable exists. The system being linear, superposition is applicable.

APPENDIX D

SCALED COMPUTER SETUP AND CORRELATION RESULTS

D.1. Scaled Layout

The REAC layout of the simulated system is shown in Figure D.1. Note that a time scale factor of 10:1 is used (i.e., response is slowed so that the computer takes 10 seconds to respond as the real system does in 1 second).

D.2. Scaled Compensator

The compensator V(s) of Chapter 3 is approximated as shown in Figure D.2. It is limited in both the inability to produce a perfect derivative and by some approximation of time constants to conform to the RC values available.

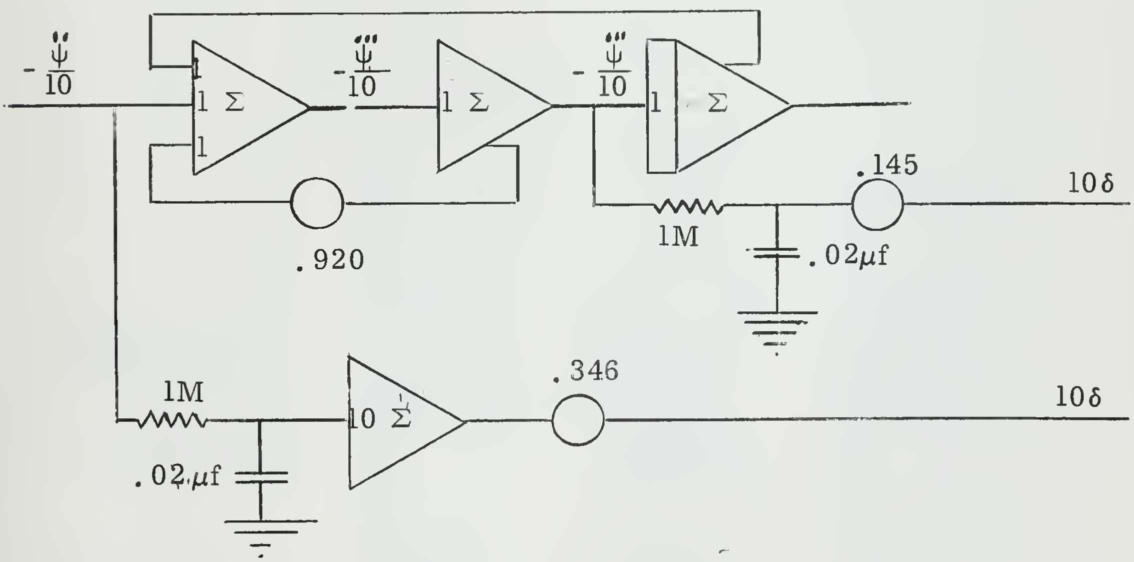

Fig. D.2 Scaled Compensator Layout

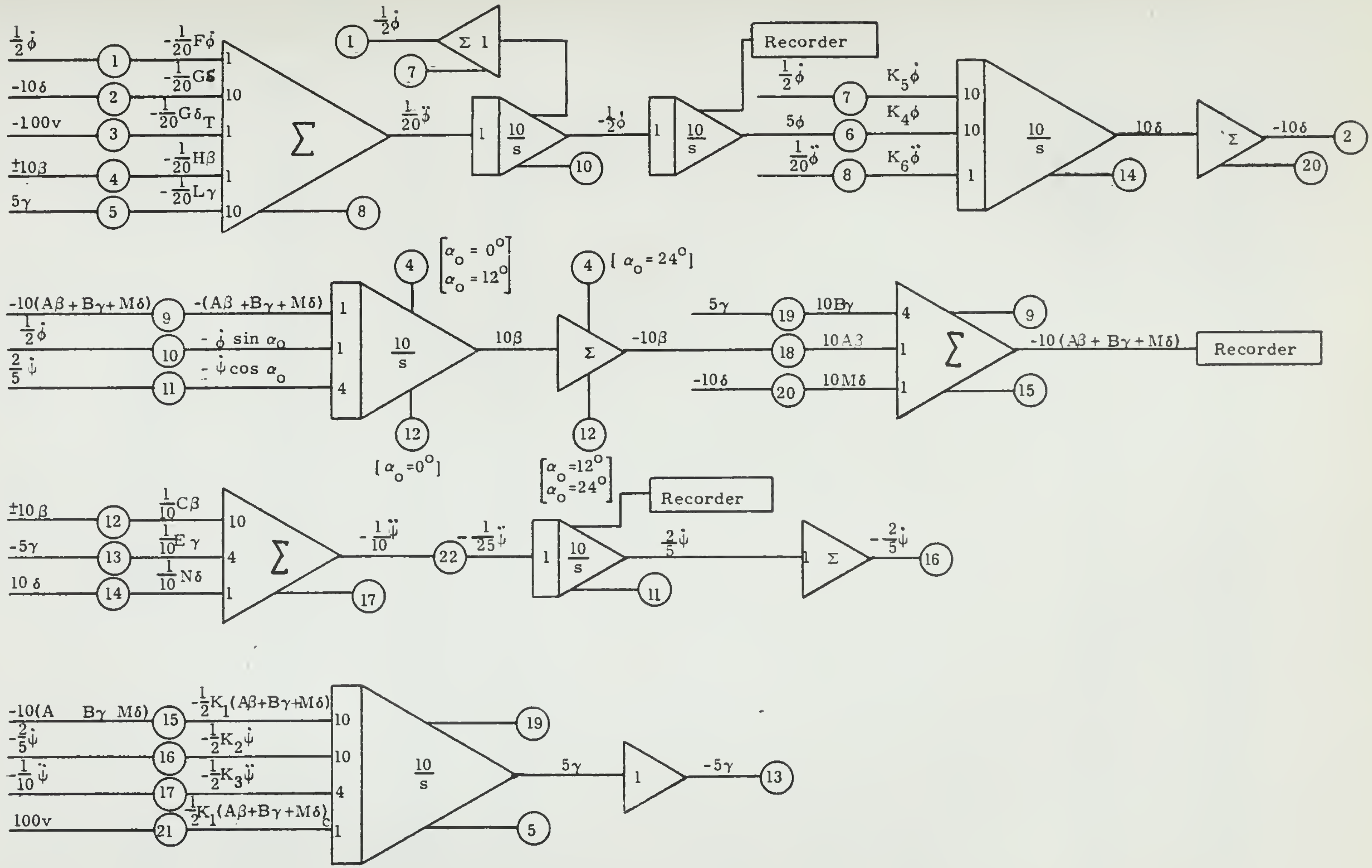

Fig. D.1 Scaled REAC Layout of Idealized System

D. 3 Response Correlation

Figures D. 3 and D. 4 are correlation check runs between computer and analytic responses of the uncoupled roll and yaw subsystems.

Gains used are not the final selected values of equations (5. 1) through (5. 6) but are listed below:

$$K_1 = 43.8 \tag{D.1}$$

$$K_2 = 6.18 \tag{D.2}$$

$$K_3 = .357 \tag{D.3}$$

$$K_4 = 27.9 \tag{D.4}$$

$$K_5 = 2.33 \tag{D.5}$$

$$K_6 = .01985 \tag{D.6}$$

When these values, analytic responses below are obtained:

$$\phi(t) = R[.464e^{-12.76t} + .465e^{-8.31t} \sin(56.28t - 85.5^{o})] \tag{D.7}$$

and

$$\dot{\psi}(t) = -P[.840 + .793e^{-3.25t} + .907e^{-7.85t} \sin(16.63t - 106.4^{o})] \tag{D.8}$$

where

$$P \equiv \left|(A\beta + B\gamma\ )_c\right| = 1\ \frac{\text{deg}}{\text{sec}} \tag{D.9}$$

and

$$R \equiv \left|\delta_T\right| = 1\ \text{deg} \tag{D.10}$$

In Figures D. 3 and D. 4, the continuous lines are computer response, circled points are check points from equations (D. 7 and (D. 8).

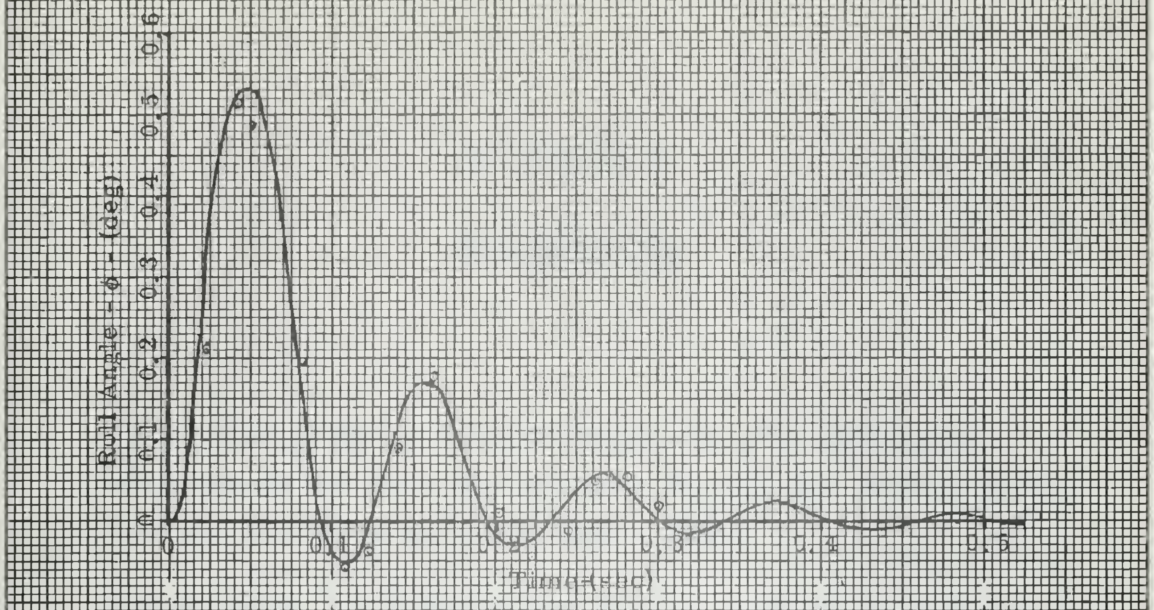

Fig. D.3 Correlation of Computer and Analytic Solutions of Roll Response

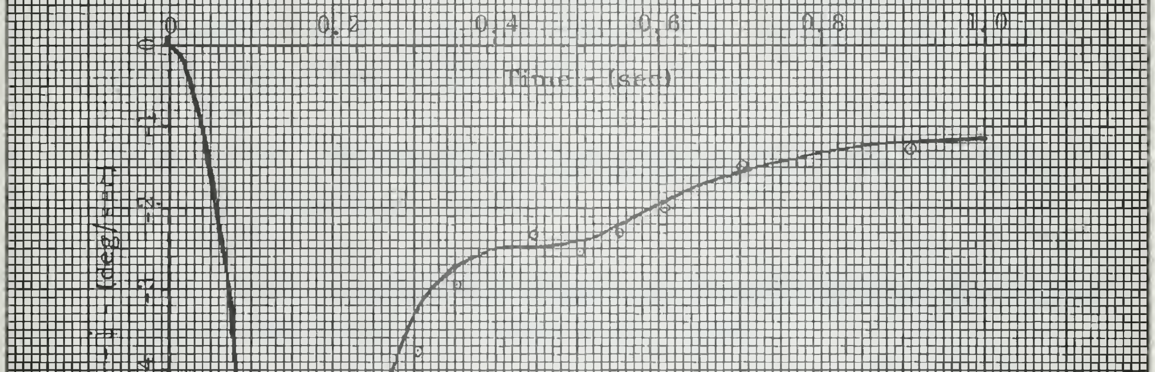

Fig. D.4 Correlation of Computer and Analytic Solutions of Yaw Rate Response

APPENDIX E

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